Comparing Grover's Quantum Search Algorithm with Classical Algorithm on Solving Satisfiability Problem

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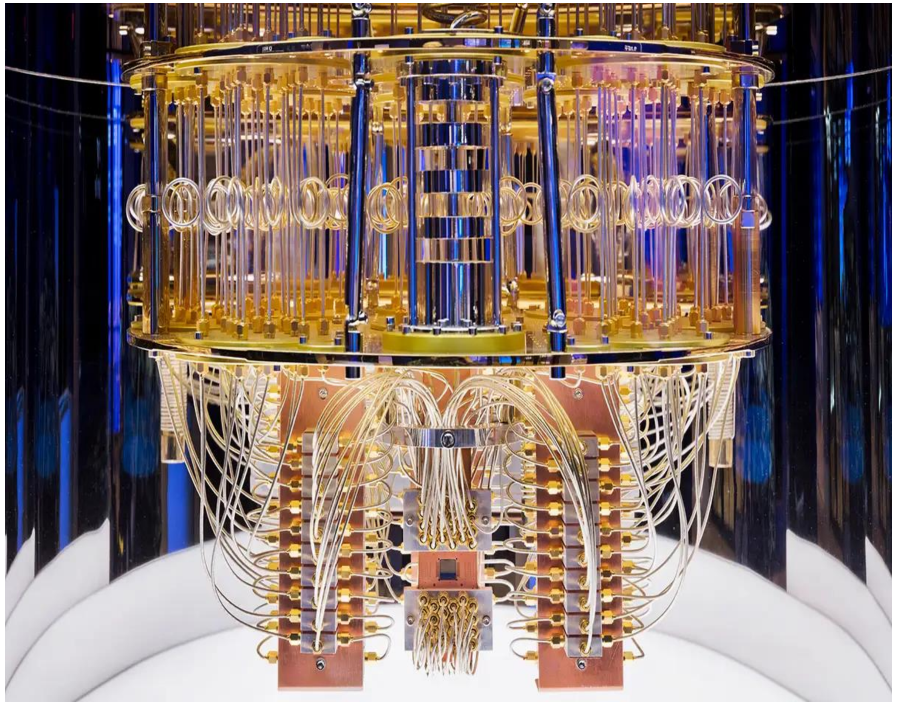
1 Introduction

1. Introduction

- **Quantum computer**
  - huge advantage
  - parallel computing ability

- **Limited number of algorithms**

- **Grover's Algorithm on database search**



1. Introduction

➢Satisfiability problem

The satisfiability problem asks the computer to find a set of values (commonly true or false) for several variables such that they satisfy certain constraints. K-SAT problems refer to satisfiability problems with k boolean variables to be determined.

➢Wide application

Combinatorial equivalence checking, automtic test pattern generation, model checking, AI planning, and haplotyping

➢Slow calculation for large cases

Traditional computation cannot handle large data well due to its exponential processing time.

1. Introduction

- 3-SAT quantum solution worse than classical solution
- k-SAT (k > 3) quantum solution unproposed
- Possible that quantum solution performs better when k grows large

1. Introduction

➢ Quantum field

- Lacks proof for its advantage

➢ Satisfiability problem

- Improvement on processing speed

➢ Grover’s Algorithm on satisfiability problem

1. Introduction

This research aims to formulate a general quantum solution for the k-SAT problem and compare such a solution with the best classical algorithm to determine whether and when the quantum algorithm performs better on satisfiability problems.

2 Prior Work

2. Prior Work

On solving the satisfiability problem, the only quantum algorithm proposed previously is focused on the 3-SAT problem [3][4].

This proposed quantum solution utilizes Grover's algorithm searching ability and traverse through all possible answer space, amplifying the correct states that satisfy the given constraints.

This solution, however, runs even slower than the optimal 3-SAT classical solution[5]. It has a time complexity of $O(1.414^N)$ as compared with the optimal classical solution with time complexity of $O(1.307^N)$, where N is the number of constraints.

3 Grover's algorithm

3. Grover's algorithm

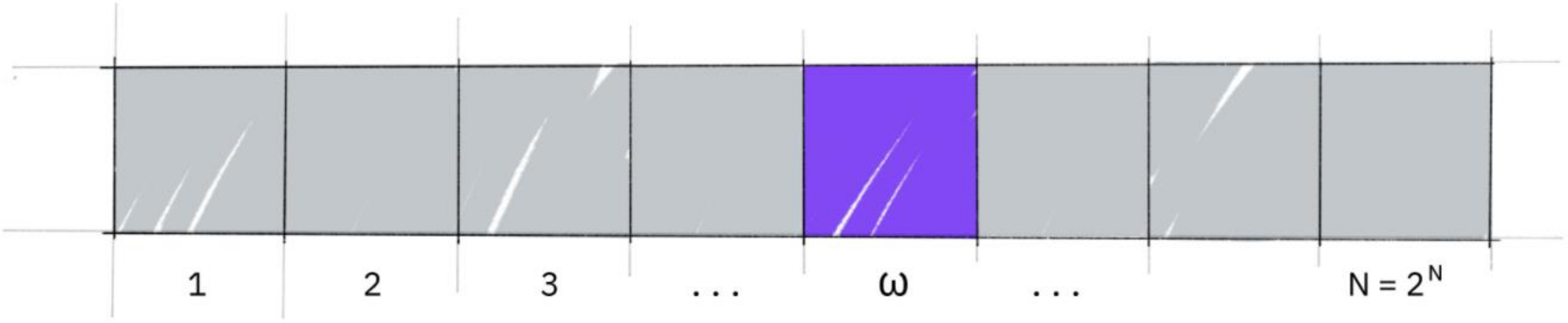
- Database search
- Traditionally: O(N)
- Grover's algorithm: $O(\sqrt{N})$

3. Grover’s algorithm

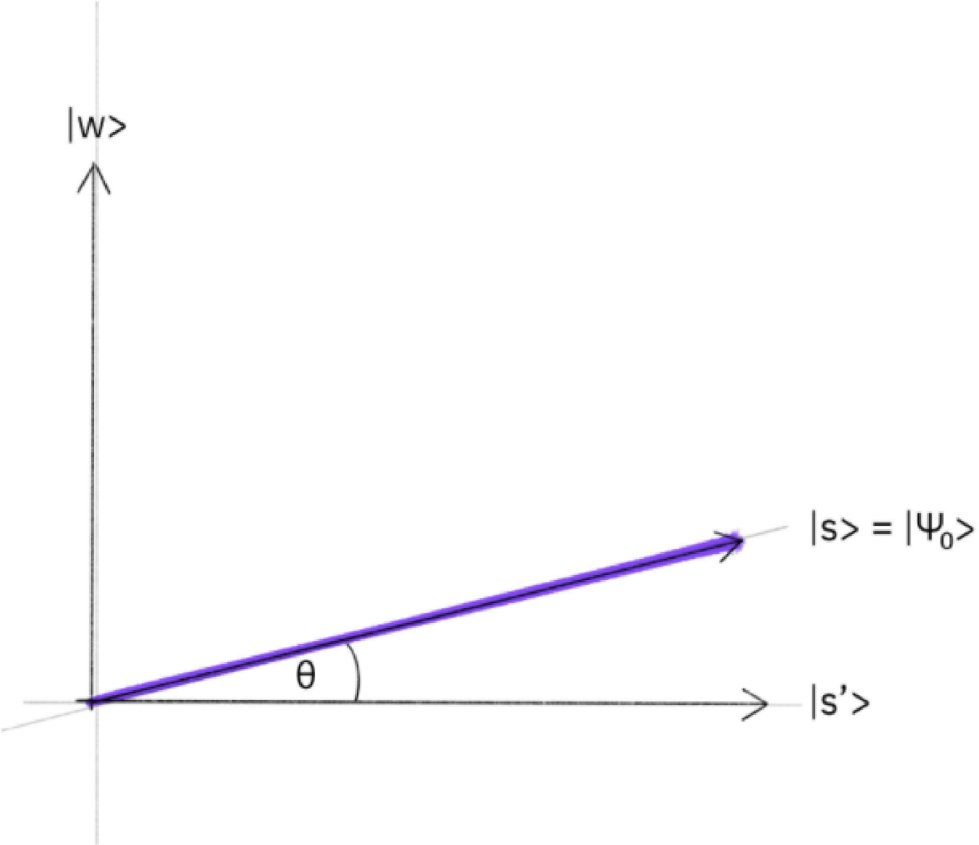
Oracle function: f(x) = {1, if x is the answer; 0, if x is not the answer}

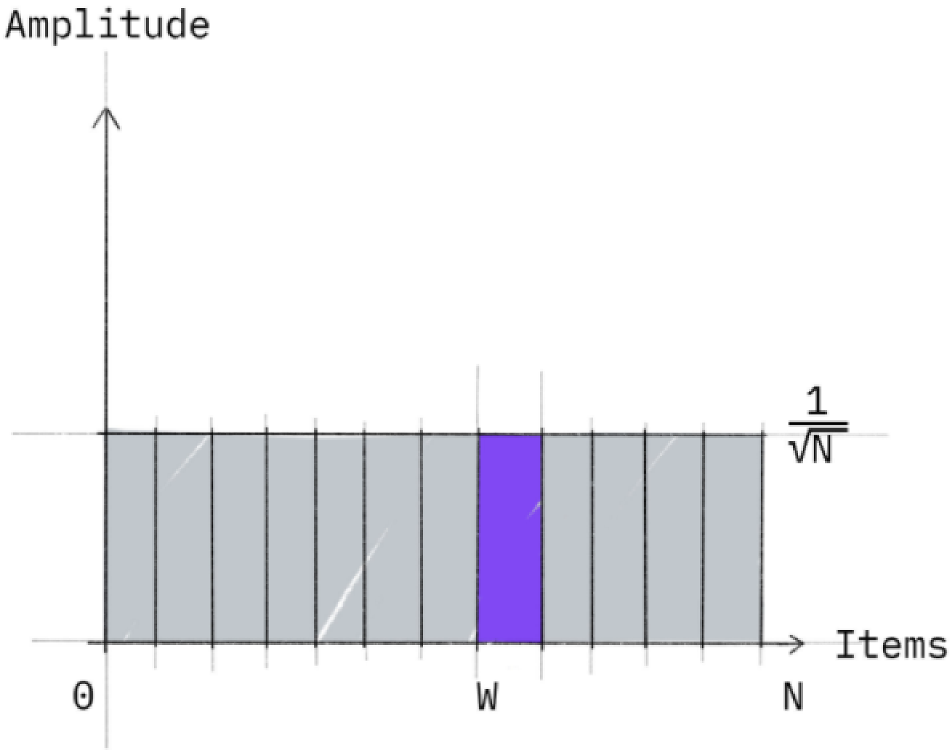
$f(\omega) = 1$

$f(1, 2, 3, \ldots, \omega - 1, \omega + 1, \ldots, N) = 0$

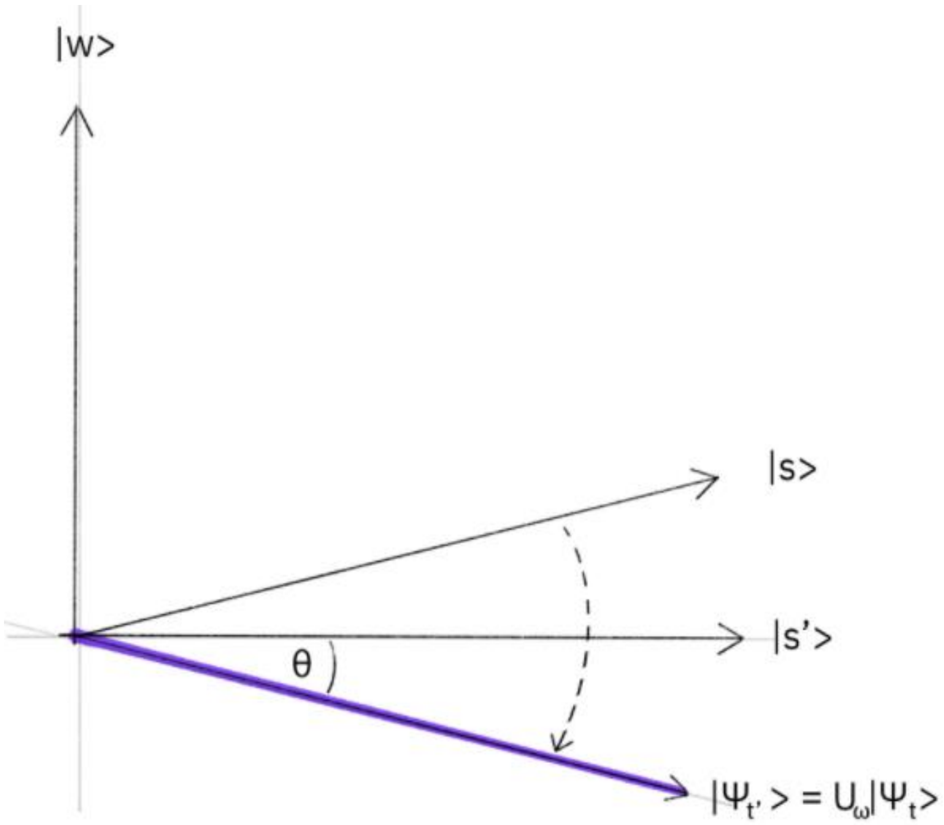


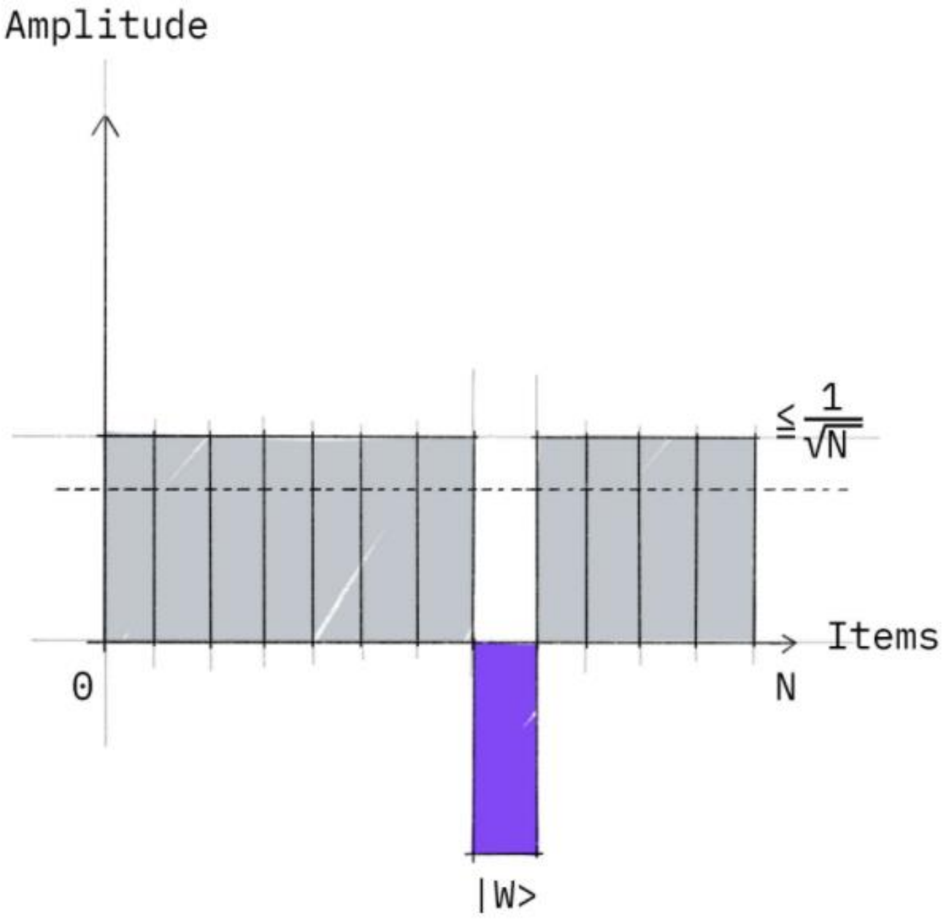
3. Grover's algorithm



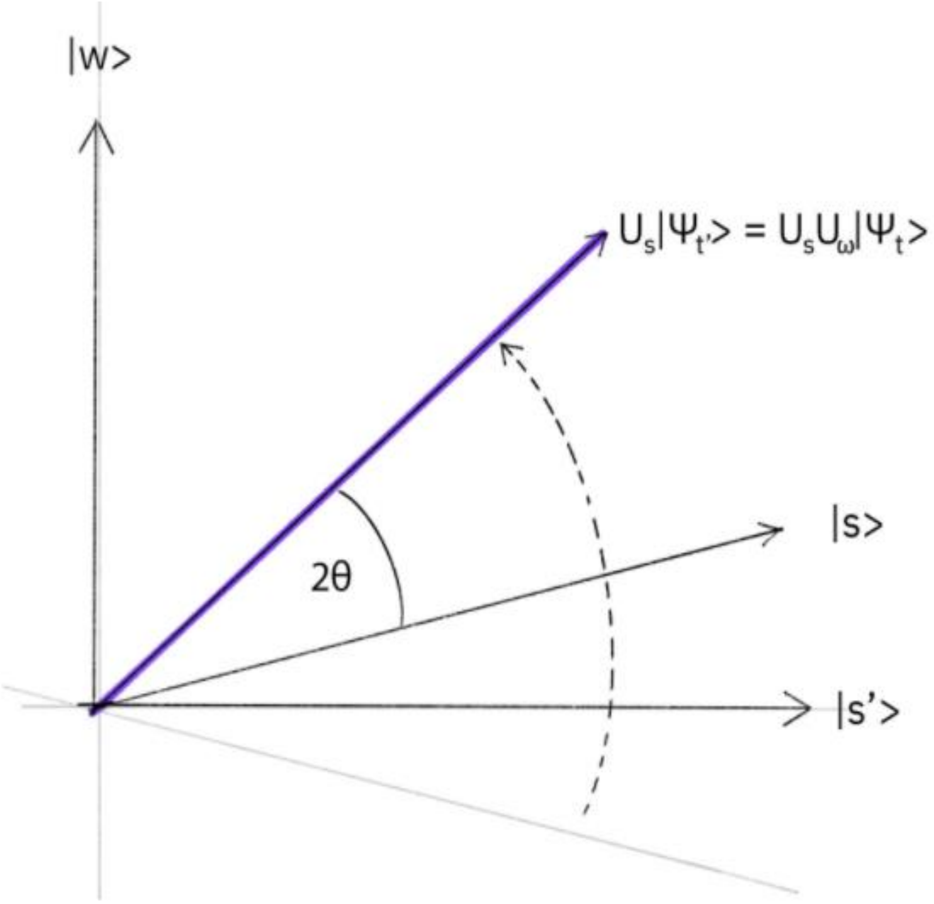


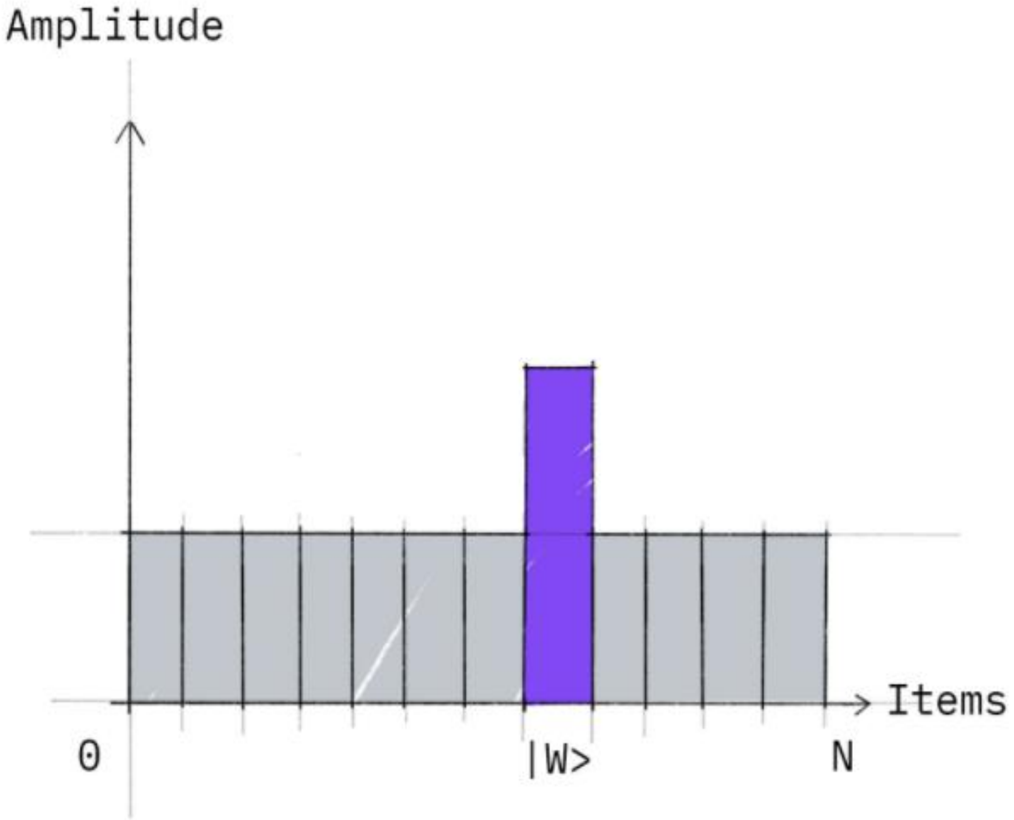
3. Grover's algorithm





3. Grover's algorithm



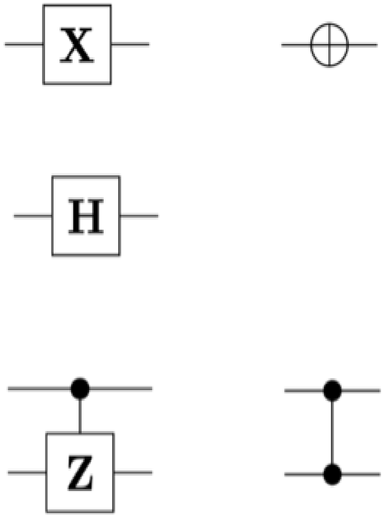


3. Grover's algorithm

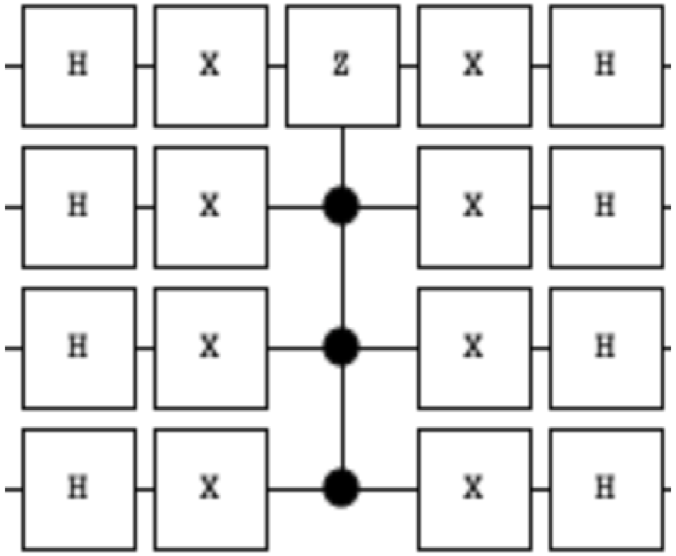
- Repeat the above process until the probability is large enough
- It can be proven that about $\sqrt{N}$ times will suffice

4 Proposed Us Circuit

4. Proposed Us Circuit



Pauli-X (X)	$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
Hadamard (H)	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
Controlled Z (CZ)	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

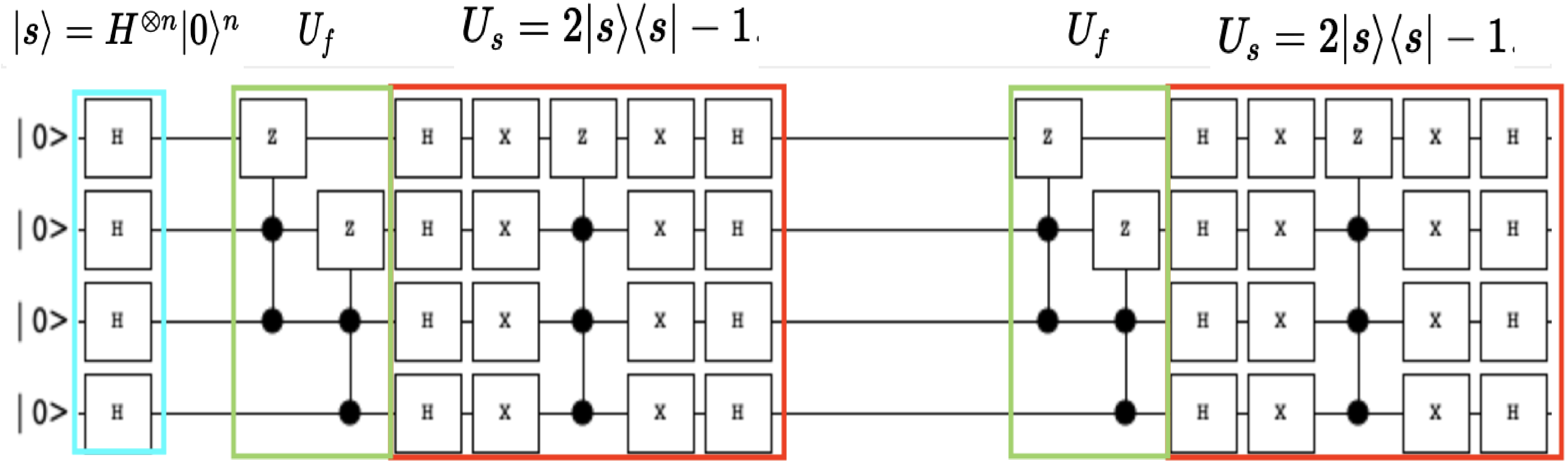


5 Testing of Us Circuit

5. Testing of Us Circuit

4-SAT example

Reverse engineered Oracle function



Simulator Result

Probability of the two correct answers is in total

94.53%

-0.06250000+0.00000000i |0000>	0.3906%
-0.06250000+0.00000000i |0001>	0.3906%
-0.06250000+0.00000000i |0010>	0.3906%
-0.06250000+0.00000000i |0011>	0.3906%
-0.06250000+0.00000000i |0100>	0.3906%
-0.06250000+0.00000000i |0101>	0.3906%
-0.06250000+0.00000000i |0110>	0.3906%
0.68750000+0.00000000i |0111>	47.2656%
-0.06250000+0.00000000i |1000>	0.3906%
-0.06250000+0.00000000i |1001>	0.3906%
-0.06250000+0.00000000i |1010>	0.3906%
-0.06250000+0.00000000i |1011>	0.3906%
-0.06250000+0.00000000i |1100>	0.3906%
-0.06250000+0.00000000i |1101>	0.3906%
0.68750000+0.00000000i |1110>	47.2656%
-0.06250000+0.00000000i |1111>	0.3906%

6 Future Work

6. Future Work

Step 1

devise Oracle function for k-SAT problem

Approach: analyse given input and construct the corresponding function

Step 2

compare the theoretical complexity with classical algorithms

- Approach: calculate complexity based on Grover's algorithm's $\sqrt{N}$ optimization
- Compare against best classical algorithm

6. Future Work

Step 3

If possible, implement on real quantum computers and real classical computers for real-world comparison

IBM quantum computer available for testing

Step 4

Analyse and summarize the result

Determine whether and when quantum algorithm outperforms classical algorithm

References

[1] D. Monroe, "Bouncing Balls and Quantum Computing," ''Communications of the ACM'', vol. 63, no. 10, Oct., pp. 10-12, 2020.
[2] Srinivasan K, Satyajit S, Behera BK, Panigrahi PK. Efficient quantum algorithm for solving travelling salesman problem: An IBM quantum experience. arXiv preprint arXiv:1805.10928. 2018 May 28.
[3] Fernandes, Diogo, and Inês Dutra. "Using Grover's search quantum algorithm to solve Boolean satisfiability problems: Part I." XRDS: Crossroads, The ACM Magazine for Students 26.1 (2019): 64-66.
[4]Fernandes, Diogo, Carla Silva, and Inês Dutra. "Using Grover's search quantum algorithm to solve Boolean satisfiability problems, part 2." XRDS: Crossroads, The ACM Magazine for Students 26.2 (2019): 68-71.
[5]Hansen, Thomas Dueholm, et al. "Faster k-sat algorithms using biased-ppsz." Proceedings of the 51st Annual ACM SIGACT Symposium on Theory of Computing. 2019.
[6]Ambainis, Andris, et al. "Quantum speedups for exponential-time dynamic programming algorithms." Proceedings of the Thirtieth Annual ACM-SIAM Symposium on Discrete Algorithms. Society for Industrial and Applied Mathematics, 2019.
[7]Srinivasan, Karthik, Bikash K. Behera, and Prasanta K. Panigrahi. "Solving linear systems of equations by gaussian elimination method using Grover's search algorithm: an IBM quantum experience." arXiv preprint arXiv:1801.00778 (2018).
[8]Kwiat, P. G., et al. "Grover's search algorithm: an optical approach." Journal of Modern Optics 47.2-3 (2000): 257-266.
[9]Ashley Montanaro, "Quantum algorithms: an overview," Nature, Jan. 12, 2016. [Online]. Available: https://www.nature.com/articles/npjqi201523. [Accessed Oct.7, 2020]

Any Questions?