The Influence of Self Interacting Dark Matter on Dynamical Friction Time Scale and the Final Parsec Problem

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The canonical cold dark matter paradigm has been shown to have significant phenomenological flaws leading to the "core-cusp" problem. It has been noticed that allowing the dark matter to self interact resolves this problem. In this paper we explore other consequences of the Self Interacting Dark Matter (SIDM) model. In particular, we investigate the effects of self interaction on the dynamical friction force incurred upon an object traversing dark matter halos. We show that SIDM has the added benefit of helping resolve "last parsec problem" allowing super massive black holes to go through binary inspirals within the Hubble time. This scenario can be tested by measuring SMBH inspiral rates at space based gravitational wave detectors such as LISA.

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I. INTRODUCTION

There is overwhelming evidence for the existence of dark matter (DM) from measurements over multiple cosmological scales [1]. Galactic measurements of rotations curves necessitate some form of DM [1] and cosmological measurements find that baryons only represent 5 percent of the mass in the universe and the dark energy is known not be account for the remaining 95 percent. The canonical cold dark matter (CDM) model assumes that the DM only interacts gravitationally, but this scenario predicts overly dense cores in the centers of galaxies and clusters [2] which has been called the "core-cusp" problem. More recently, it has been conjectured that allowing the DM to self interact can solve this problem [12]. Such dark matter is called Self Interacting Dark Matter (SIDM). This paper explores how self interactions affect the mechanism of dynamical friction and its phenomenological consequences. We concentrate on how SIDM affects the inspiral of Super Massive Black Hole (SMBH) binary inspirals, where a direct way to check for SIDM can be made via the measurement of the inspiral rate for SMBH binaries, which are a primary target of space based gravitational interferometers such as LISA [3].

A. The Dynamical Friction in a Collisionless Medium

Dynamical friction is a mechanism for energy loss that plays a crucial roll in the merger of astronomical objects such as quasars, globular clusters and SMBHs. It is defined as the the gravitational drag force on a body travelling through the field of a gravitating medium such as a star cluster or dark matter. Typically one assumes the medium to be collisionless, but the effects of collisions can be significant, as we will explore below. Dynamical friction is caused by the gravitational force induced by the wake which accumulates behind the perturber (a SMBH in our case). When calculating the inspiral rate for a SMBH merger, the time scale for the inspiral based upon this force is called the "dynamical friction time scale" which must be shorter than the Hubble scale for the phenomena of interest to be phenomenologically relevant.

The dynamical friction force for a mass traveling through a collisionless medium was first calculated by Chandrasehkar [4] for a perturber interacting with a field of stars. He consid-

[1] In princinple one could try to change gravity at long distances but at present no consistent model has been found.

ered the effects of individual collisions with fixed impact parameter b and then intergrated over the range of impact parameters. Assuming a Maxwellian distribution for the medium it was found that

$$\frac{d\vec{v}}{dt} = -\frac{4\pi G^2 M_p \rho(r) ln\Lambda}{(v)^3}[erf(X) - \frac{2X}{\sqrt{\pi}}e^{-(X^2)}]\vec{v} \tag{1}$$

where $Erf[X]$ is the error function of X, v is the velocity of the perturber, M_p is the mass of the perturber, ρ(r) is the mass density of the medium at radius r, X is the $\frac{v}{\sqrt{2}\sigma}$ and σ is the velocity dispersion of the stars in the galaxy such that $\sqrt{2}\sigma \equiv v_c$. The logarithm is called the "Coulomb Log" and is due to the long range nature of the gravitational force and is given by

$$\ln \Lambda = \ln \frac{b_{max}}{b_{90}} \tag{2}$$

where b_{max} is the maximum impact parameter, approximately the orbital radius of the subject body and b_{90} is the 90 degree reflection radius, the impact parameter for which the deflection angle of the reduced particle of the encounter is equal to 90 degrees. Notice that b_{max} is in general unknown, but an order of magnitude estimate suffices in general. The frictional acceleration is proportional to M_p so that the frictional force is proportional to ${M_p}^2$. This is due to the fact that the magnitude of the wake that accumulates behind the perturber is proportional to M_p.

During the decay, the radius of the perturber is decreasing very slowly compared to the tangential velocity. Therefore, the perturber is moving in a nearly circular orbit with velocity equal to the orbital velocity. For a certain $X = v/V_c$,where V_c is the circular orbit speed at radius r the angular momentum loss is then given by

$$\frac{dL}{dt} = f \times r \tag{3}$$

where f is the dynamical friction force. The isothermal condition of the medium, implies that $v \sim v_c$, so that $X = 1$. Using the fact that rotation curves are flat we have

$$\rho(r) = \frac{v_c^2}{4\pi G r^2} \tag{4}$$

along with eq.(1) and (3) we can write the differential equation:

$$\frac{\mathrm{d}L_\mathrm{S}}{\mathrm{d}t} = r\frac{dv}{\mathrm{d}t} = -0.428 \ln \Lambda \frac{GM_p}{r} \tag{5}$$

Given the nearly circular nature of the orbit we have $L = rv_c$, where v is approximately constant. Plugging this relation into (5)

$$r\frac{dr}{dt} = \times 0.428 \ln\Lambda \frac{GM_p}{v_c} \tag{6}$$

Given that Λ is approximately constant through the decay we can calculate the coalescence time t_c by terminating t when $r = 0$ [7]:

$$t_c = \frac{1.65 v_c r_i^2}{\ln\Lambda G M_p} \tag{7}$$

where r_i is the initial capture radius. For future use we re-write this scale as

$$t_c = \left(\frac{4 \times 10^6\ yr}{LogN^\star}\right)\left(\frac{\sigma_c}{200\ km/sec}\right)\left(\frac{r_i}{100pc}\right)^2\left(\frac{10^8 M_\odot}{M_p}\right) \tag{8}$$

where $N_\star$ is the number of stars in the core.

The other relevant time scale during a binary inspiral is the time scale for gravitational radiation. That is, it is the time it takes for a coalescence assuming the only energy loss mechanism is gravitational wave radiation, i.e. ignoring dynamical friction. Using the leading order quadrapole radiation formula it can be shown that [3]

$$\tau_{\rm GW} = \simeq 2.9 \times 10^6 {\rm yr}\left(\frac{r_i}{0.01{\rm pc}}\right)^4\left(\frac{10^8 M_\odot}{M_p}\right)^3. \tag{9}$$

B. The Last Parsec Problem

Given that we have two mechanisms for energy loss in SMBH binaries we may ask what is the event rate for SMBH binary decays at LISA? The signal to noise ratio is expected to be large enough for LISA to detect such events for a wide range of masses and red-shifts. For binaries with masses in the range $10^4 M_\odot < M < 10^8 M_\odot$ one can detect binaries for redshifts up to around 10 [5] with varying signal to noise ratios. At present the event rate is unknown and depends upon the nature of the dynamical friction force. If we assume a collsionless medium then equation (8) applies. Note however that this result assumes that the inspiralling mass is moving slower than the typical velocity of the medium (e.g. the stars in a star field). Once the orbit shrinks sufficiently, the relative velocity becomes very large due to conservation f angular momentum. And sine the dynamical friction force is proportional to $\frac{1}{v^2}$, there are little appreciable interactions so that the dynamical friction mechanism shuts off. If this shut off time occurs before gravitational wave emission becomes effective then the inspiral will not occur in a Hubble time and the event becomes irrelevant to LISA. We can estimate the radius for this shut off occurs to be

$$r_{SO} \approx \frac{GM_P}{4\sigma_c^2} = 3.5pc\left(\frac{180km/s}{\sigma_c}\right)^2\left(\frac{M_p}{10^8 M_\odot}\right). \tag{10}$$

where we use a reference velocity of $180km/sec$ as this is the typical value for stars surrounding SMBH of the mass $\sim 10^8 M_\odot$. Plugging in this radius into (9) we find $t_{GW} \sim 10^{17}$ years, which is 10^7 times the age of the universe. Thus, SMBH inspirals only become a viable event candidate if there is some mechanism to reduce r_i in (9). This is known as the "last parsec problem" (for a recent update see [8]). N-body simulations indicate that for triaxial galaxies the gap in time scales shrinks and the problem is resolved but for other morphologies this remains an open problem.

C. Dynamical Friction and Self Interacting Dark Matter

As discussed above, SIDM has been proposed to solve the cusp-core problem, so it is natural to ask if it can have other phenomenological consequences. In particular, we would like to investigate what effects SIDM have on SMBH binary inspirals. First we must reconsider the dynamical friction mechanism within the context of a collisional gas. Here we utilize the analytic study of collisional friction in [9] where one no longer considers collisions with a fixed impact parameter but instead treats the dark matter as an adiabatic medium in a gravitational potential. Since dark matter is collisional it supports sound waves with velocity c_s.

When the inspiralling BH is moving slower than the DM sound speed, the subsonic case, the dynamical friction force is given by [9]

$$f = \frac{4\pi(GM_p)^2\rho_0}{v^2}\cdot(\frac{1}{2}ln\frac{1+M}{1-M} - M) \tag{11}$$

where ρ_0 is the local DM mass density. When the perturber is moving supersonically, the friction force is

$$f = \frac{4\pi(GM_p)^2\rho_0}{v^2}\cdot(\frac{1}{2}ln(1-\frac{1}{M^2}) + ln(\frac{vt}{R_m in})) \tag{12}$$

Where $M = v/c_s$ and v is the speed of the perturber and $R_m in$ is the effective size of the perturber. Treating the dark matter as an ideal gas, the speed of sound can calculated as

$$cs = \sqrt{\frac{\gamma RT}{m}} \tag{13}$$

γ is the adiabatic index, R is the molecular gas constant, m is the dark matter particle mass, and T is the temperature of SIDM. The mass, as well as the temperature, are unknown free parameters. If the SIDM candidate is going to compose all of the missing energy density, modulo the dark energy, then the mass will be determined by the decoupling temperature.

Here we take two extreme assumptions:

First, we assume that the mass of each particle of dark matter is 9 times the average mass of a single particle in a galaxy. Since most of the particles in a galaxy are Hydrogen and Hellium. In the milky way the ratio between hellium and hydrogen is 25:74, so that the assumed average particle mass is

$$9 \times (0.25 M_{He} + 0.74 M_H) = 11.2 \tag{14}$$

and the sound speed should be:

$$cs_1 = 54.2 m/s \tag{15}$$

Second, we assume thatthe number density is 9 times the average number density of the matter in a typical galaxy. In this case the sound speed should be:

$$cs_2 = 162.9 m/s \tag{16}$$

This shows that the sound speed in sidm can vary greatly for different models of SIDM particles, but they genereally stays in the same order of magnitudes.

There are assumptions between two extremes. For example 3 times the number density and 3 times the average mass density, which result in the sound speed between 54.2 m/s and 162.9 m/s.

Given a dark matter desnity and sound speed we may use equations (11) and (12), to determine the dynamical friction time scale, for both subsonic and supersonic conditions, in a fashion similar to what was done above, to determine the effect of the self interacting nature of the DM.

II. ANALYSIS

A. The dynamical friction time scale and SIDM

for different dark matter particle models, we found the following relationship between the sound speed and atom mass.

Assume that the temperature of such SIDM is the same(2.725K),

$$cs = \sqrt{\frac{\gamma RT}{M}} \tag{17}$$

For SIDM with average particle mass between 0.000124 (0.1 times the average particle mass celestial matter) to 0.0124 (10 times the average particle mass celestial matter), the sound speed of dark matter varies as:

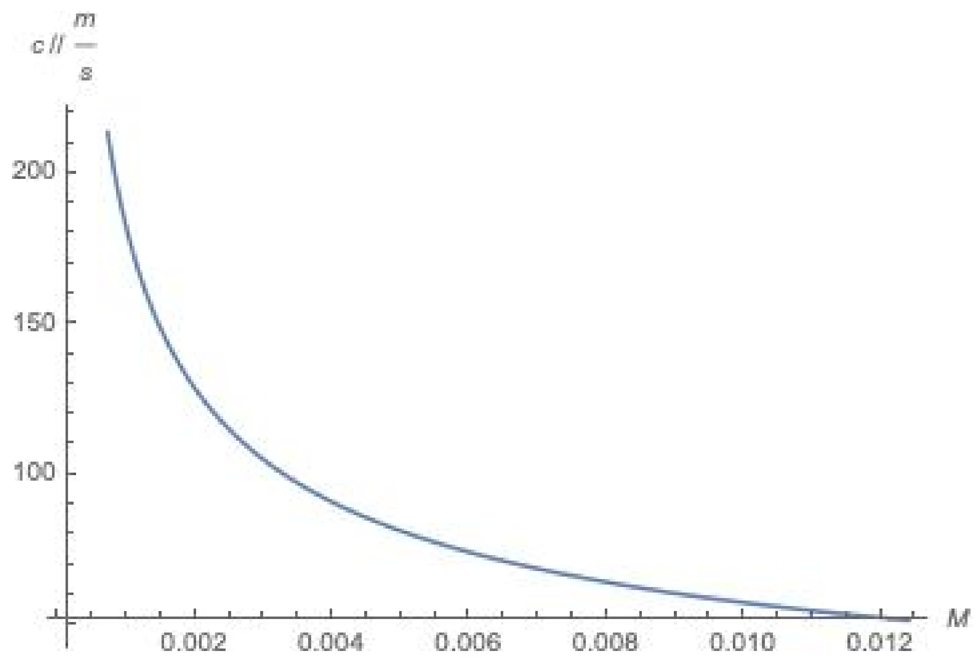

FIG. 1: Sound speed in dark matter

In the case of a subsonic perturber with velocity v tranvsersing a SIDM medium the specific angular momentum ($L^* = r \times v$) loss is given by

$$v\frac{dr}{dt} = r \times \frac{4\pi(GM_p)^2\rho_0}{v^2} \cdot (\frac{1}{2}ln\frac{1+M}{1-M} - M) \tag{18}$$

Since $M = v/c_s$ changes slowly on the orbital time scale during the entire process, we may again use the relationship

$$\frac{4\pi G\rho(r)}{v^2} = \frac{1}{r^2} \tag{19}$$

so that we can write

$$r\,dr = \frac{GM_p(\frac{1}{2}ln\frac{1+M}{1-M} - M)}{v}dt \tag{20}$$

taking the vecloty to be approximately constant and integrating leaves

$$t_{c1} = \frac{r_i^2 v}{2GM_p(\frac{1}{2}ln\frac{1+M}{1-M} - M)} \tag{21}$$

For a supersonic perturber, the angular momentum loss rate is

$$v \times \frac{dr}{dt} = r \times \frac{4\pi(GM_p)^2\rho_0}{v^2} \cdot (\frac{1}{2}ln(1 - \frac{1}{M^2}) + ln(\frac{vt}{R_{min}})) \tag{22}$$

The integration of time:

$$\int \frac{GM_p}{v}(\frac{1}{2}\ln(1 - \frac{1}{M^2}) + \ln(\frac{vt}{r_{min}}))\,dt = \int r\,dr \tag{23}$$

And the integration result in

$$\frac{1}{2}r_i^2 = \frac{GM_p}{v}(\frac{1}{2})\ln(1 - \frac{1}{M^2})t + t\ln(\frac{v}{R_{min}}t) - t \tag{24}$$

Below we will solve for t numerically.

Here we have used the finite integration that

$$\int_{t_i}^{t_f} \ln(t)\,dt = (t\ln(t) - t)\,|_{t_i}^{t_f} \tag{25}$$

III. RESULTS

We would like to compared our results with the traditional result without considering dark matter dynamical friction in eq. (8). We have used some typical choices for parameters: for the galactic radius we take $r = 10^{20}m$ while for the central mass $M_H = 10^{45}$ and perturber mass is chosen to be $M_p = 10^{40}kg$, which were approximately $10^{15} and 10^{10}$ Solar mass. In figure (1) we show the ratio of the dynamical friction forces (SIDM/baryonic) as a function of time in the subsonic case. For the supersonic condition case we can see from (10) that we need to input R_{min} so we choose the typical value $R_{min} = 10^{20}$ with all other parameters the same. We again approximate the velocity to be $v = V_c$, to remain constant during the decay. In figure (2) we show the same ratio as in figure one for the case of supersonic decay. We can see that in both cases SIDM can improve the situation leading to the solution of the last parsec problem.

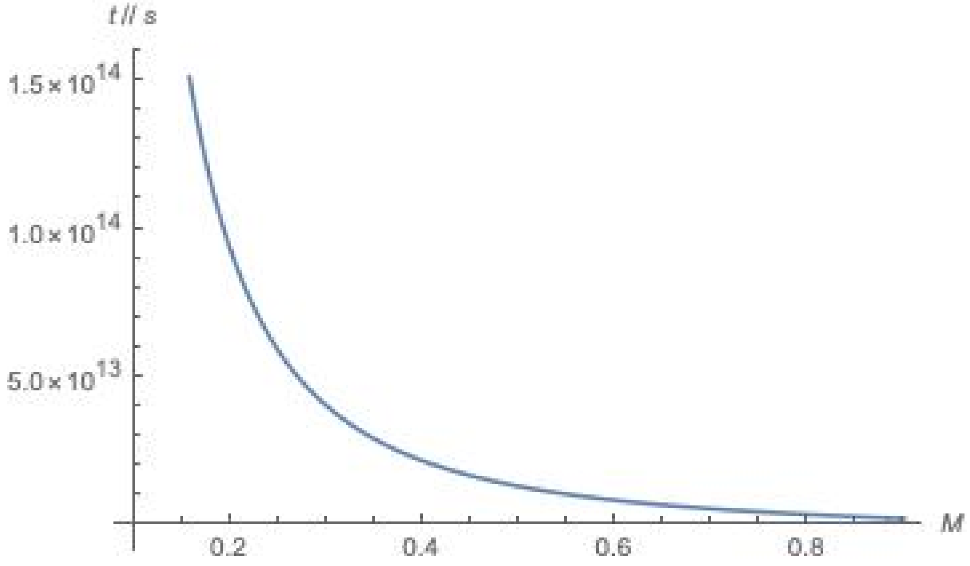

FIG. 2: SIDM dynamical friction subsonic condition

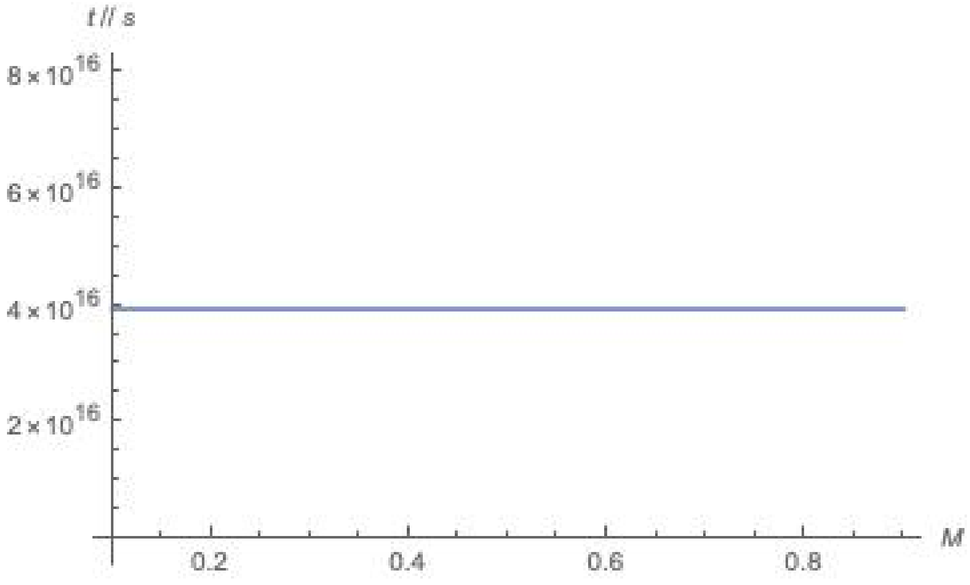

FIG. 3: Baryonic dynamical friction subsonic condition

we also plotted the radius the perturbers able to decay in time t for subsonic condition

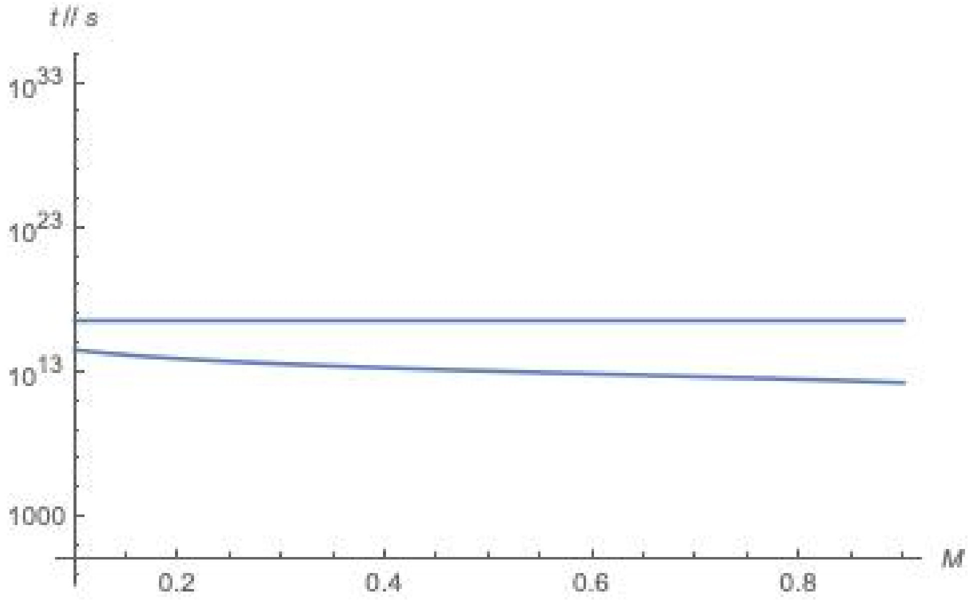

FIG. 4: Dynamical friction time scale ratio for the subsonic condition

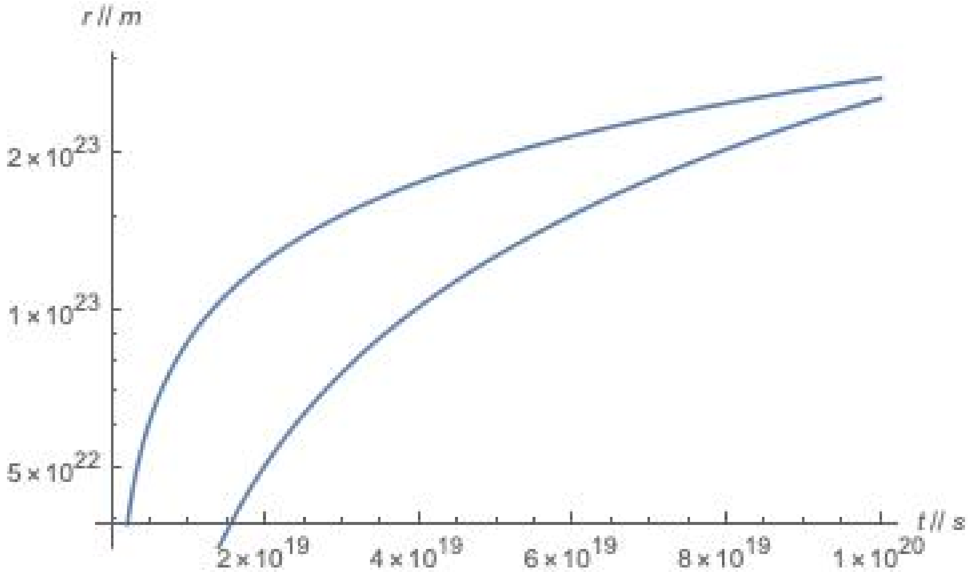

FIG. 5: Dynamical friction time scale r-t ratio

The result shows that, for such a galaxy, the smaller the galaxy is, the more effect does dark matter dynamical friction has on the coalesce time. The larger the galaxy is the more effect does traditional dynamical friction it has on the coalesce time. The point of return, shown below, is 10^{28} m.

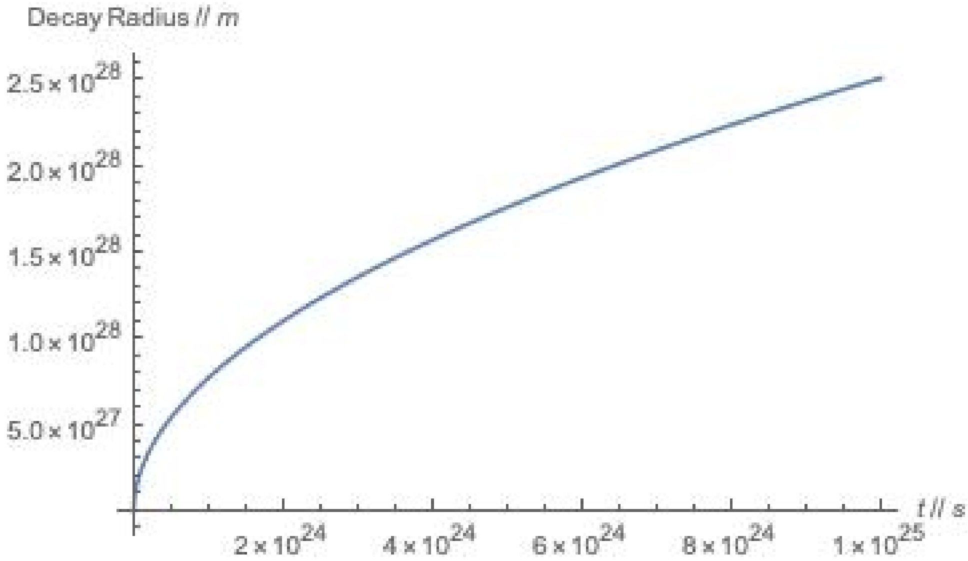

FIG. 6: SIDM dynamical friction supersonic condition

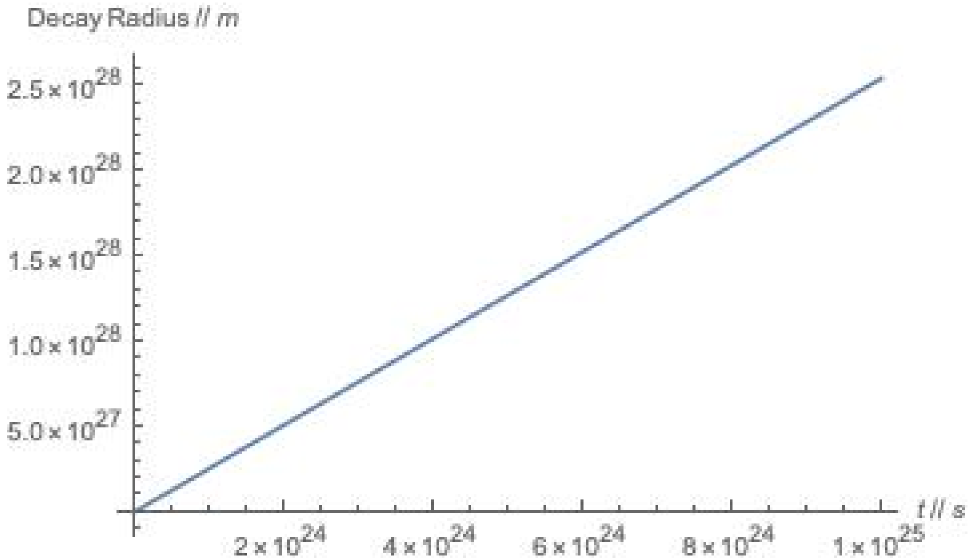

FIG. 7: Baryonic dynamical friction supersonic condition

A. Other Bounds on SIDM

We are not free to choose our SIDM parameters arbitrarily. There are astrophysical [11] and cosmological constraints [10] on the SIDM. Assuming that dark matter has the same temperature as background radiation, there exists a bound on dark matter particle mass $m > 70$ MeV[10]. Therefore the sound speed in dark matter should not exceed.

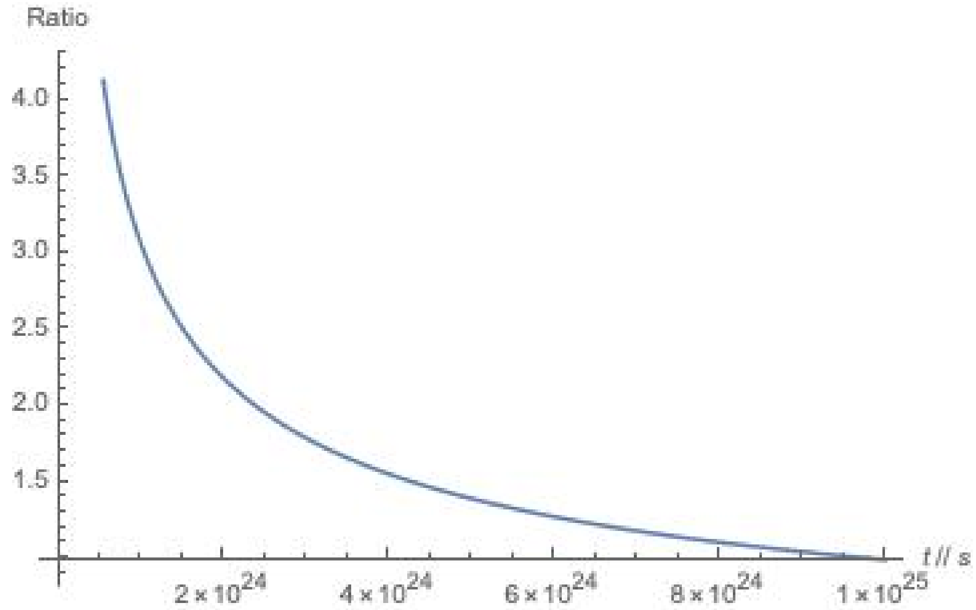

FIG. 8: Ratio for supersonic condition

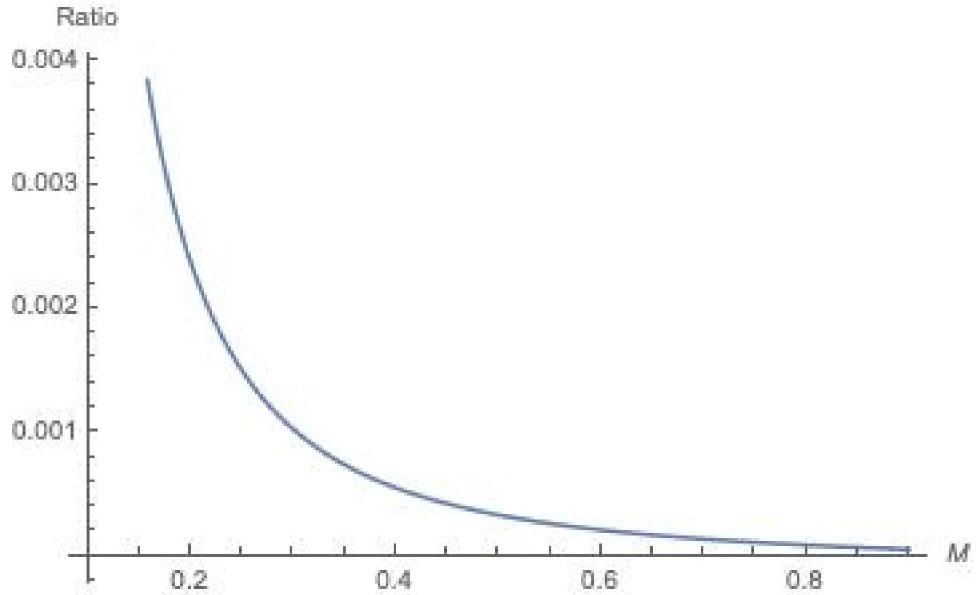

FIG. 9: Ratio for subsonic condition. Ratio between dynamical friction timescales due only to baryonic friction and SIDM friction.

$$c_s^{max} = \sqrt{\frac{\gamma R \times 2.2725K}{70MeV/c^2 \cdot m_p}} = 664m/s. \tag{26}$$

B. Dark matter models

Here is the 3d graph about how different assumption of average dark matter particle mass affect the result

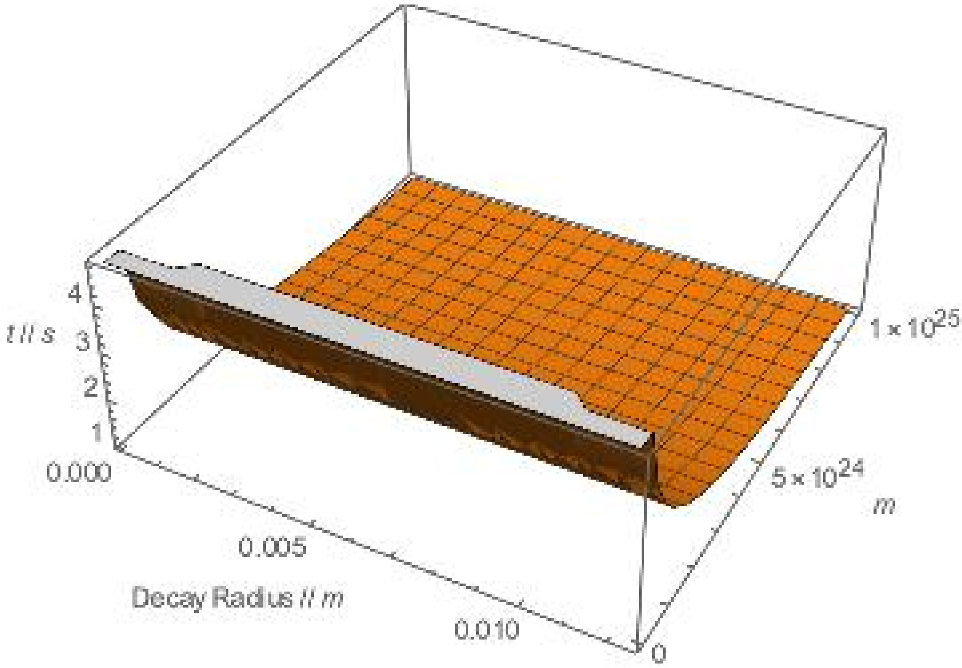

FIG. 10: Subsonic condition dark matter model influence

For subsonic condition, the average mass of dark matter particle affects the ratio greatly, which means the smaller average mass of dark matter particle is the bigger effect does traditional dynamical friction has on coalesce time.

For the supersonic condition, the average mass of dark matter particle has little affect on the coalesce time.

IV. CONCLUSION

The nature of dark matter is still a mystery. Direct detection has yet to identify a candidate, thus we must rely on cosmological measurements to bound its properties. The standard cold dark matter model is believed to lead to discrepencies with the matter distibutions in galaxies. It has been suggested that allowing the dark matter to self interact, which seems like a natural possibility, can improve agreement with data. Here we have investigated the effect of SIDM on the binary inspirals of super massive black holes. In particular, we have

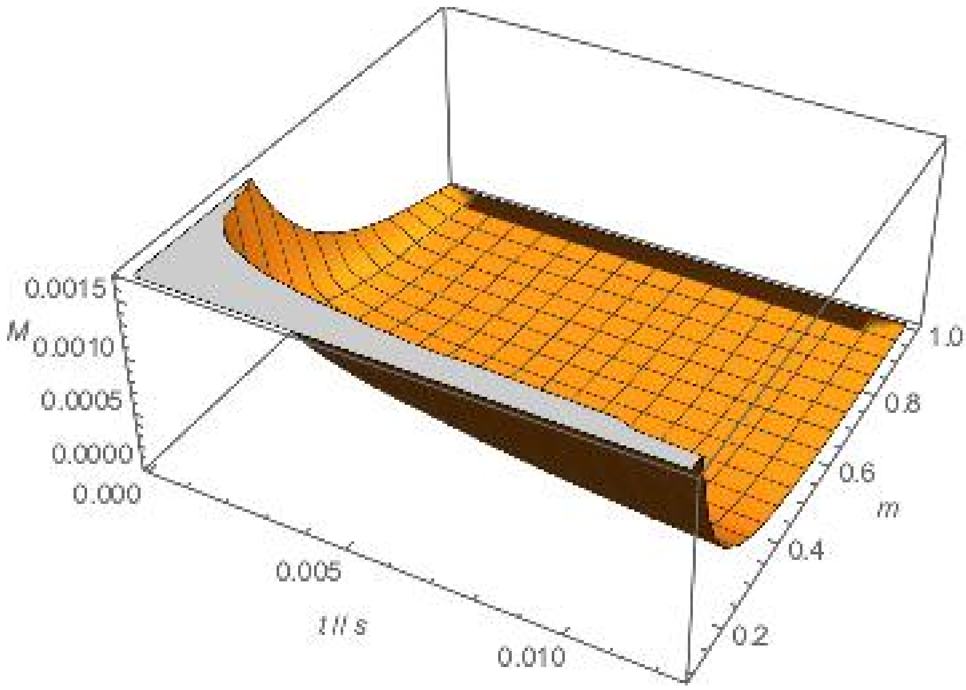

FIG. 11: Supersonic condition dark matter model influence

shown that SIDM can reduce the time scale for merger leading to a higher signal rate in space based gravitational wave detectors. In particular we have shown that for black hole velocities less than the speed of sound in dark matter SIDM can reduce to time scale for the collapse of the binary inspiral.

For different models of dark matter particles, only in the subsonic condition does smaller average mass of dark matter particles greatly decrease the influence of dark matter dynamical friction.

These conclusions may help explain some observations that are greatly biased with traditional dynamical friction calculation. In the assumption that dark matter can exert dynamical friction on merging SMBH, the force may help solving the last parsec problem by decreasing the final separation for the SMBH binary. Binaries may reach a separation smaller than a few parsec that help the binary to merge faster than in previous calculation.

V. FUTURE DIRECTIONS

The possibility of SIDM leads to other avenues of exploration in the context of dynamical friction.

In the assumption that dark matter can exert dynamical friction on merging SMBH, the force may help solving the last parsec problem by decreasing the final separation for the SMBH binary. However, how dark matter behave under relativistic condition is still under discussion, and it can be very different from what we have discussed in the previous paragraphs.

Here we have only pointed out the leading order effect but further study is necessary. In particular, an exploration of the parameters space, varying the masses as well as sound speeds will allow us to determine the parameter space expected at LISA. The research used the background temperature 2.725 K as the temperature of dark matter particles, but this temperature may vary. For SIDM, the temperature should be the same. However for theories that dark matter does no interact with baryons in any form, the temperature should be discussed further. Also, since we know little about dark matter, it is very likely that it does not follow the sound speed equation for baryons. Moreover, here we have assumed that the the sound in dark matter is a long lived coherent oscillation. However, one could imagine that a finite viscosity in the dark matter could affect the dynamical friction time scale. Such a viscosity would have other phenomenological consequences at multiple length scales.

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