Using Mathematical Modeling Tackling the Drought

Project ID: 375
Category: Environmental Science and Engineering
Division: Senior

Introduction

Background: Because of the Climate Change, the Western and Southern parts of the United States are under severe drought. This is leading to the decreasing in Volume of Lakes and shortage of water all around the States.

Fig 1: The drought data in the week from March 1st to March 8th from National Integrated Drought Information System [1].

Challenges:

- Drought is a complex phenomenon that is difficult to monitor.
- There is limited technology to predict the appearance of Drought.

Research Question:

- Is there any effective and practical way to help scientists predict the coming or the period of (Hydrological) drought?

Research Task:

- Using one specific water resource suffering the drought as an example to build the general model of drought predicting.

Experiment Steps

Material preparation: Choosing the experiment subject: Mead Lake, a Colorado River reservoir on the Nevada-Arizona border, is the largest water reservoir in the United States. In the summer of 2021, Lake Mead registered its lowest level on record since its initial filling in the 1930s [2].

Step 1: Set up the function of the volume of water in Lake Mead.

Step 2: Modeling the water level and elevation of Lake Mead.

Step 3: Modeling and planning of wastewater Recycling.

Step 4: Providing Plans about water recycling

Step 5: Making conclusion and summary the future work

Volume model

Comprised of three parts:
Total Volume = Original Volume + Inflow - outflow - Loss

- Inflow: Water from three sub-streams
  - Measure the contour line of the water surface and bottom develop the function
  - Find the area of cross section by doing Integral of function of the difference between surface and bottom line
  - Using volume formula, using the cross-sectional area times the velocity of the sub-streams

- Outflow: Water released by Dam and directly consumed in Lake

- Loss: Water loss mainly from the evaporation
  - Simplifying Penman's formula and apply it to the Lake

$$E_0 = \frac{700T_{\mathrm{m}}/(100 - L_a) + 15\,(T - T_{\mathrm{d}})}{(80 - T)}\ (\mathrm{mm\ day}^{\ -1})$$

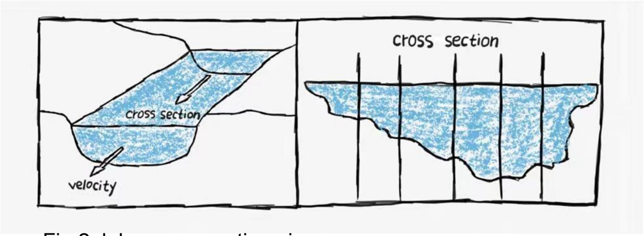


Fig 2: lake cross section view

$$S = \int_{x_1}^{x_2} f_{up}(x) - f_{down}(x)dx$$

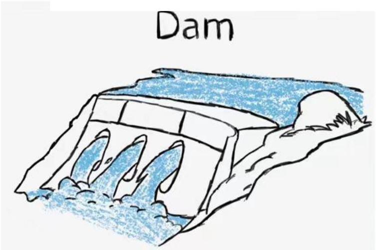


Fig 3: Dam

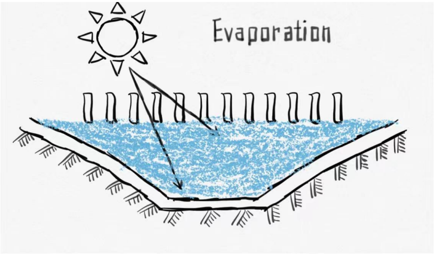
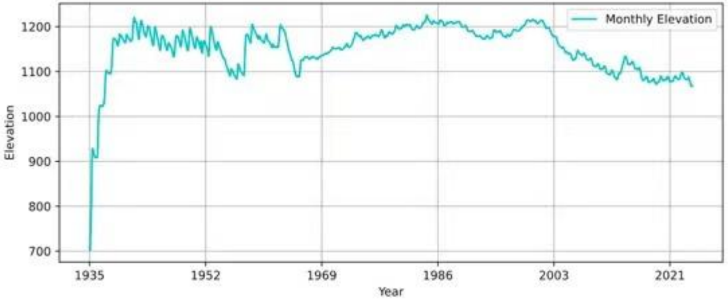


Fig 4: Evaporation

Notation: All the drawing figures in this page is drawn by myself

Water Level Prediction - Overall Patterns

- Analysis the historical volume level since 1939



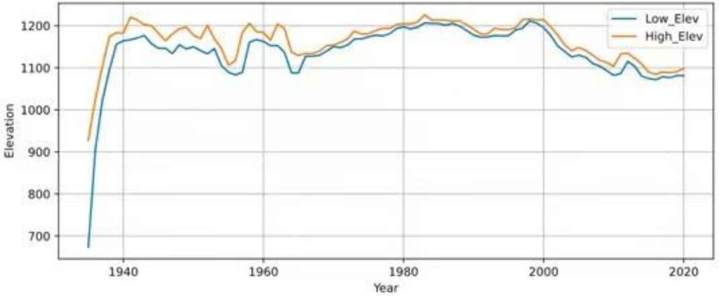
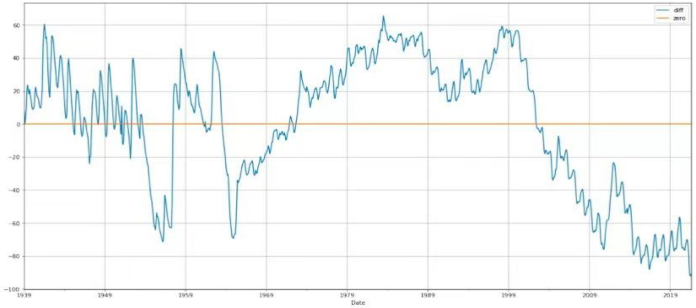


Fig 5: Left: Monthly Water Elevation.

Right: The Highest and Lowest Water Elevation.



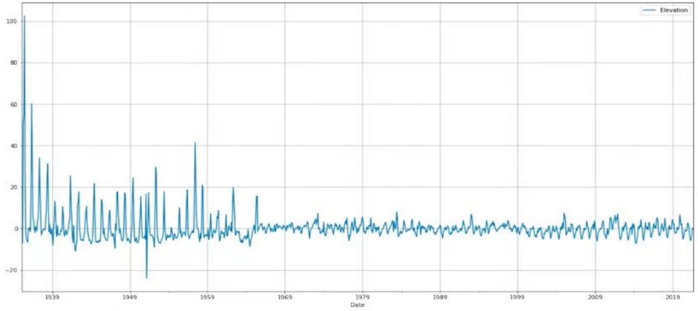


Fig 6: Left: Water Level After 1939.

Right: the Water Level After Difference.

Notation: All the figures in this page is created by myself

Water Level Prediction - Overall Patterns

Based on the figures, the following Patterns:

- Since 1935, the water level of Lake Mead began to rise sharply to 1150 feet.
- After the water level became stable, it fluctuated between 1100 feet and 1200 feet.
- The water level of Lake Mead fluctuates seasonally.
- After year 2000, the water level of Lake Mead declined in a fluctuation.
- The annual maximum water level and minimum water level of Lake Mead fluctuate synchronously.

According to the Patterns, Long Term Time Series and Short Term Time Series models are built and will finally be compared using the predicting data.

Water Level Prediction - Building Short Term Model

1. Collecting the Water Level in recent three years

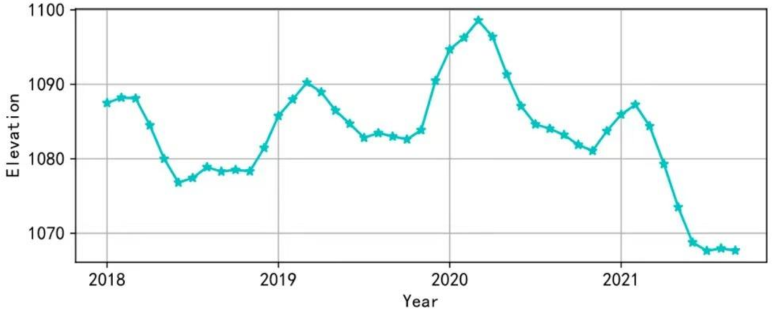
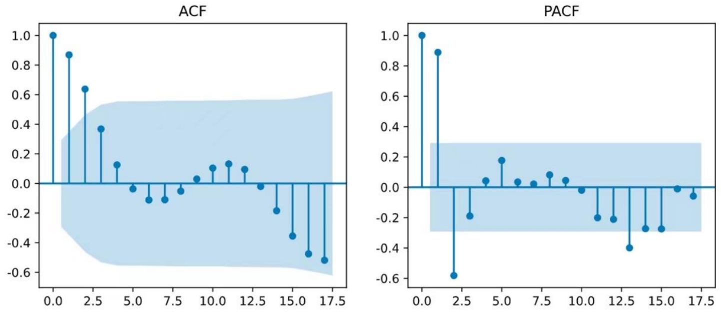


Fig 7

2. Graphing auto-correlation Function (ACF) and partial (PACF) to choose Model



Notation: All the figures in this page is created by myself

Fig 8

3. Using the chosen Model (ARMA(2,4)) to do the prediction

ARMA Model Results

Dep. Variable:	y	No. Observations:	45
Model:	ARMA(2, 4)	Log Likelihood	-89.786
Method:	css-mle	S.D. of innovations	1.700
Date:	Sun, 14 Nov 2021	AIC	195.573
Time:	16:34:01	BIC	210.026
Sample:	0	HQIC	200.961

	coef	std err	z	P>|z|	[0.025	0.975]
const	1082.6772	2.773	390.388	0.000	1077.242	1088.113
ar.L1.y	1.4516	0.547	2.655	0.008	0.380	2.523
ar.L2.y	-0.5735	0.430	-1.333	0.182	-1.416	0.269
ma.L1.y	0.4209	0.568	0.741	0.459	-0.693	1.535
ma.L2.y	0.0538	0.617	0.087	0.931	-1.155	1.262
ma.L3.y	0.0287	0.451	0.064	0.949	-0.855	0.913
ma.L4.y	-0.1530	0.279	-0.548	0.583	-0.700	0.394

Fig 9

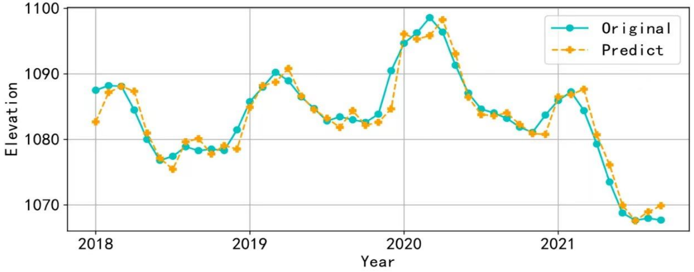


Fig 10

Water Level Prediction - Building Long Term Model

1. Collecting the Water Level since 2005

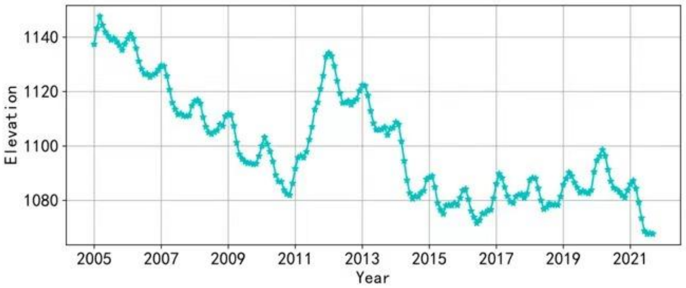


Fig 11

2. Because of Linear trend, doing first order difference and apply ARIMA model

3. Using the chosen Model (ARIMA(3,1,2)) to do the prediction

ARIMA Model Results

Dep. Variable:	D.y	No. Observations:	200
Model:	ARIMA(3, 1, 2)	Log Likelihood	-404.184
Method:	css-mle	S.D. of innovations	1.787
Date:	Sun, 14 Nov 2021	AIC	822.369
Time:	17:13:43	BIC	845.457
Sample:	1	HQIC	831.712

	coef	std err	z	P>|z|	[0.025	0.975]
const	-0.3122	0.410	-0.761	0.447	-1.116	0.492
ar.L1.D.y	1.6626	0.056	29.823	0.000	1.553	1.772
ar.L2.D.y	-1.6544	0.057	-28.879	0.000	-1.767	-1.542
ar.L3.D.y	0.6608	0.055	11.987	0.000	0.553	0.769
ma.L1.D.y	-0.9150	0.024	-38.539	0.000	-0.962	-0.868
ma.L2.D.y	0.9998	0.031	32.632	0.000	0.940	1.060

Fig 12

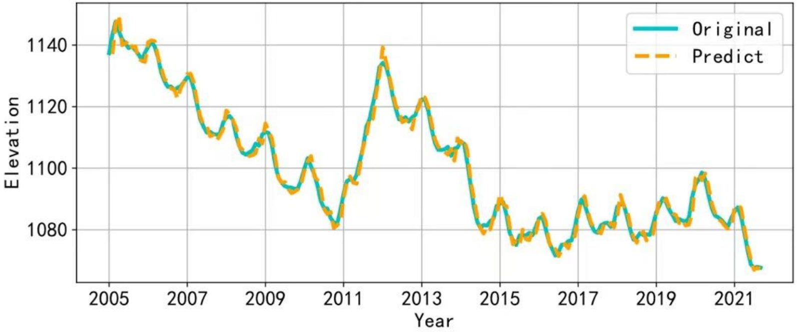


Fig 13

Notation: All the figures in this page is created by myself

Water Level Prediction - Result Comparison

- Short Term Time Series Model predicts that the annual average water level in 2025, 2030 and 2050 will be 1082.40, 1082.37, 1082.31 respectively, basically maintained near 1080, which meets our earlier expectation.

- Our LTTS Model predicts that the annual average water level in 2025, 2030 and 2050 will be 1050.84, 1029.09, 942.10 respectively, which shows a significant descending trend and meets our earlier expectation.

- Compared to STTS Model, LTTS Model has a more accurate prediction because more history data are used.

Future Water Usage and Waste Water Recycling - Factors

Based on our models and water level predictions in Section 6, the water level of Lake Mead will be less than 1000 feet in the long run. Thus, it is quite important to recycle wastewater.

Factors in Recycling the Water

- Industrial Wastewater Recycle
- Domestic Wastewater Recycle
- Commercial Wastewater Recycle
- Surface Wastewater Recycle such as Rainstorm
- Oil and Other Wastewater Recycle

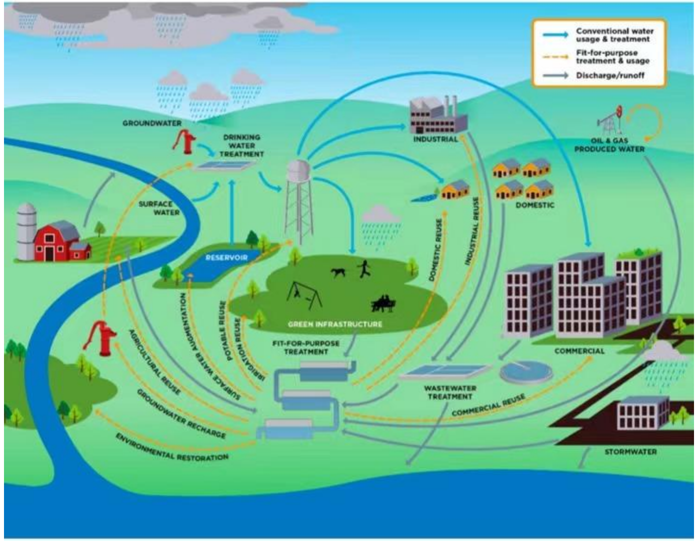


Fig 14: The example of recycling process[2]

Future Water Usage and Waste Water Recycling - Plans

- Enterprises are required to comply with regulatory requirements such as adopt wastewater treatment facilities to meet strict pretreatment or direct discharge standards.

- Reduce or eliminate wastewater treatment surcharges.

- Reuse water. For the treated water that can be reused, the factory will recover high-quality effluent to reduce the operation cost of the factory.

- Convert wastewater into green energy. Valuable biogas can be produced from wastewater through anaerobic digestion, which has become a cost-saving renewable energy.

Conclusion

Even though the result from prediction model are really expected there are still some weakness.

In the process of modeling and analysis, we assumed that the annual precipitation is constant

The amount of water directly consumed from the lake is also constant each year.

Although these quantities are uncertain, they rely on more information in actual situations. It should be possible to portray them better.

In the future, we are looking forward to containing more factors that could contribute to the change of the volume of the lake and further generalize a common model that could be used to all the Lakes.

Reference

[1]Google search. https://www.drought.gov/current-conditions#:~:text=As%20of%20March%201%2C%202022,48%20states%20are%20in%20drought.&text=of%20the%20U.S.%20and%2059.17,are%20in%20drought%20this%20week.&text=acres%20of%20crops%20in%20U.S.%20are%20experiencing%20drought%20conditions%20this%20week.

[2]Google Seaarch.https://www.comap.com/highschool/contests/himcm/2021_Problems/2021_HiMCM_Problem_B.pdf

[3]Google search.https://mathinsight.org/media/image/image/double-integral\-volume\-under\-surface\-box.png