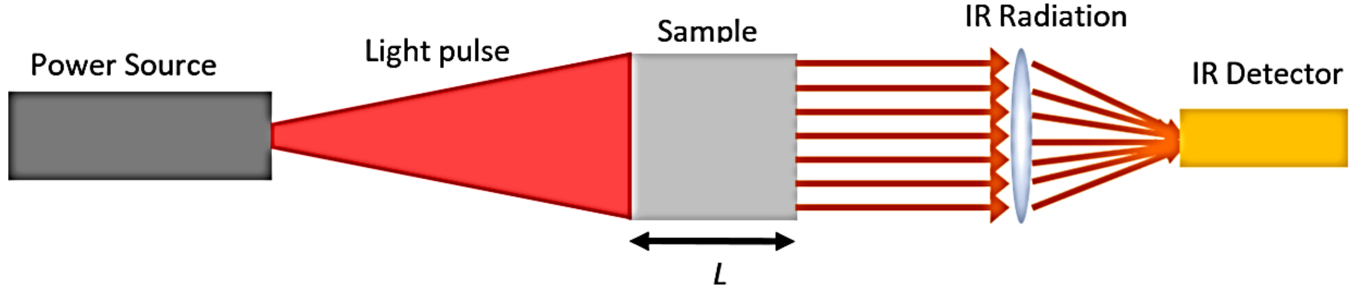
# Motivation

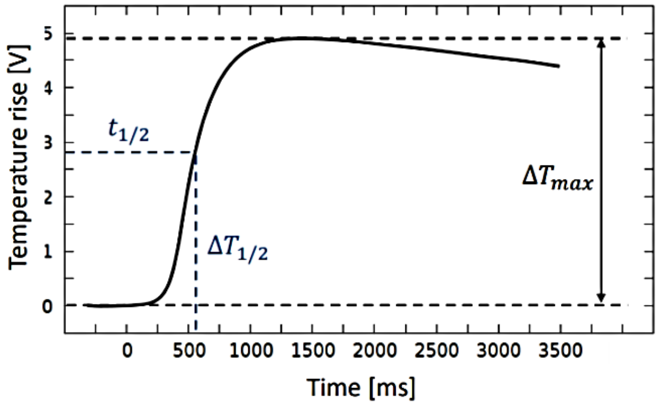
- Thermal diffusivity is an important material parameter which is essential to understand materials' thermal properties and further in materials' various applications.
- Being able to accurately measure the thermal diffusivity of a material is important.
- A simple and low cost measurement method is thus highly desirable.

# The Existing Research Results

- Thermal diffusivity is often measured with the flash method. It involves heating a strip or cylindrical sample with a short energy (such as, laser) pulse at one end and analyzing the temperature change (reduction in amplitude and phase shift of the pulse) a short distance away.
- The method requires more complex process and expensive instruments.
- Other alternative methods also involve expensive and complex equipment.




Laser Flash Method scheme

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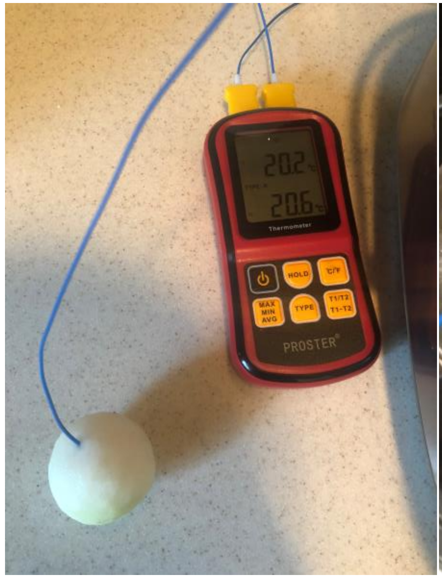
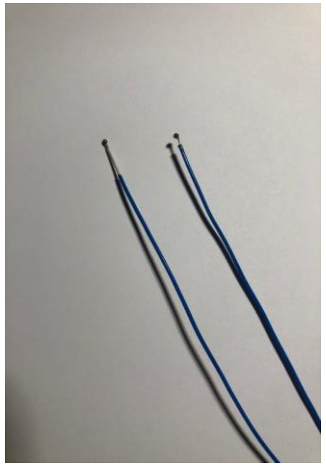
# This research: A simple, low-cost, fast, and accurate method to measure the thermal diffusivity of various foods

- ✓ Simple: samples are very easy to prepare.
- ✓ Low-cost: the total cost for the materials, tools, and instruments used in this research is less than $200!
    - Thermal couple and electronics: $60.
    - Caliper: $20
    - All the food materials: $50.
    - Other containers and cooking wares: $50.
- ✓ Fast: it takes less than 30 minutes to measure each sample.
- ✓ Accurate: the measured results are consistent and accurate.


# Temperature Measurement



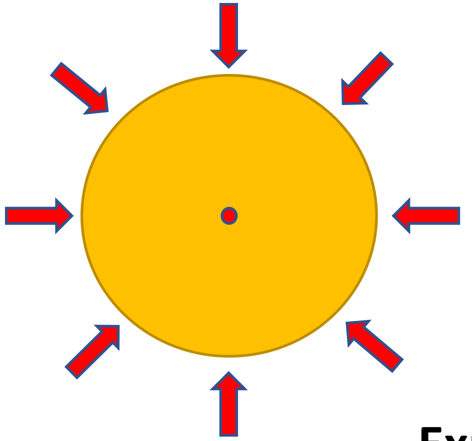
- ✓ Need to measure temperature at a special location. The temperature sensor head needs to be small.
- ✓ Need to track the temperature as a function of time.
- ✓ A thermo-couple device is used. The thermo-couple has a small size with a diameter around 1 mm and the thickness of the plastic cloth is also about 1 mm in diameter.
- ✓ The thermo-couple is inserted into the center of the spherical shaped food sample to measure the temperature rising as the function of time during the heating process.

# Experimental Process

- ❑ Thermal couples with small diameters are used to measure the center temperature of the samples.
- ❑ The samples are boiled in boiling water (100 C).
- ❑ Track the center temperature rise as the function of time.

# Sample Preparation

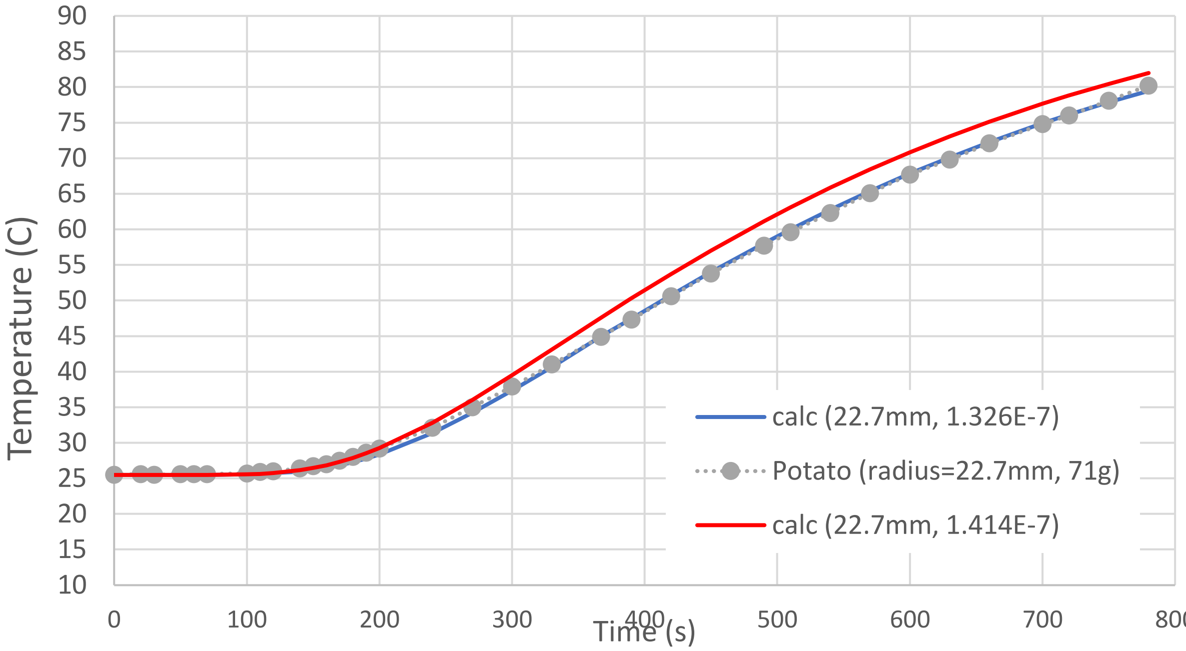
- ✓ Different food samples are carefully cut into nearly perfect spheres with different diameters (left: sweet potato, center: potato, right: taro).
- ✓ Why sphere? See below.



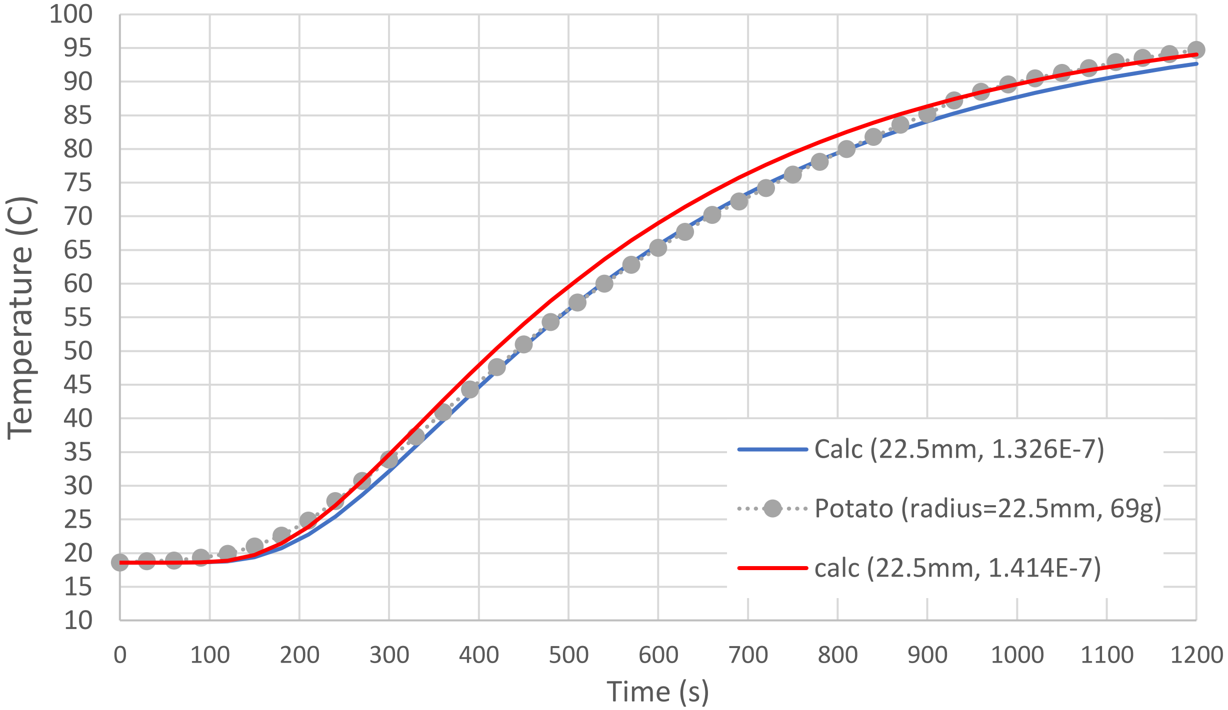
# Why Spherical Shape?

- ❑ Spherical symmetry makes the distance from the center only “parameter”.
- ❑ Theoretically, it is easy to simulate.
- ❑ Experimentally, it is easy to measure.
- ❑ The comparison between the theoretical calculation and the experimental measurement become possible and straightforward.

## Experimental Measurement and Theoretical Simulation (fitting)

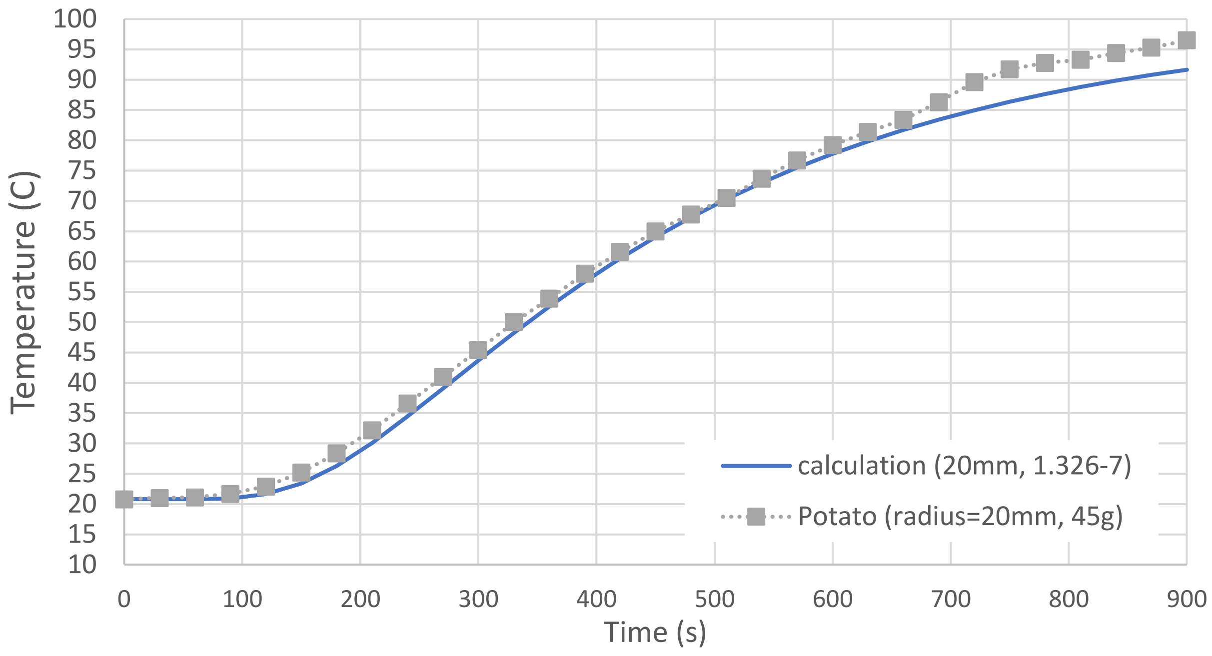


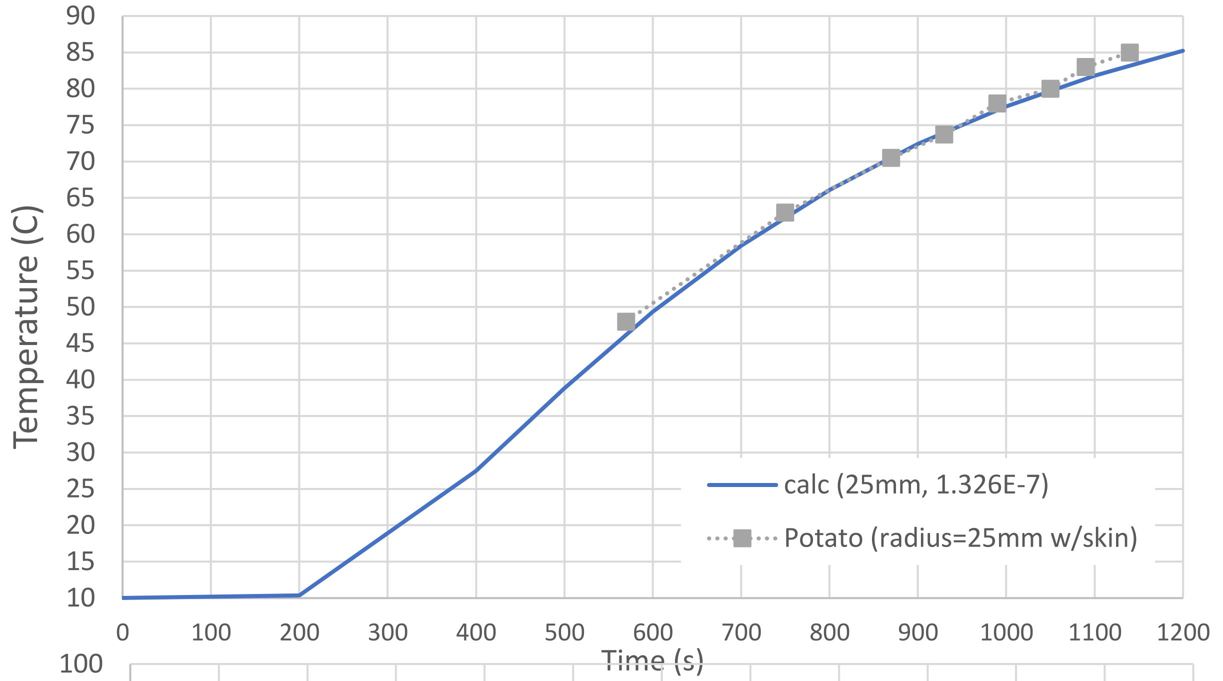

A potato sample with a diameter of 22.7mm was measured. The temperature of the center of the sphere sample is recorded as the function of time as shown in dots. The solid curves are calculated results based on the thermal diffusivity data as indicated in the figure.

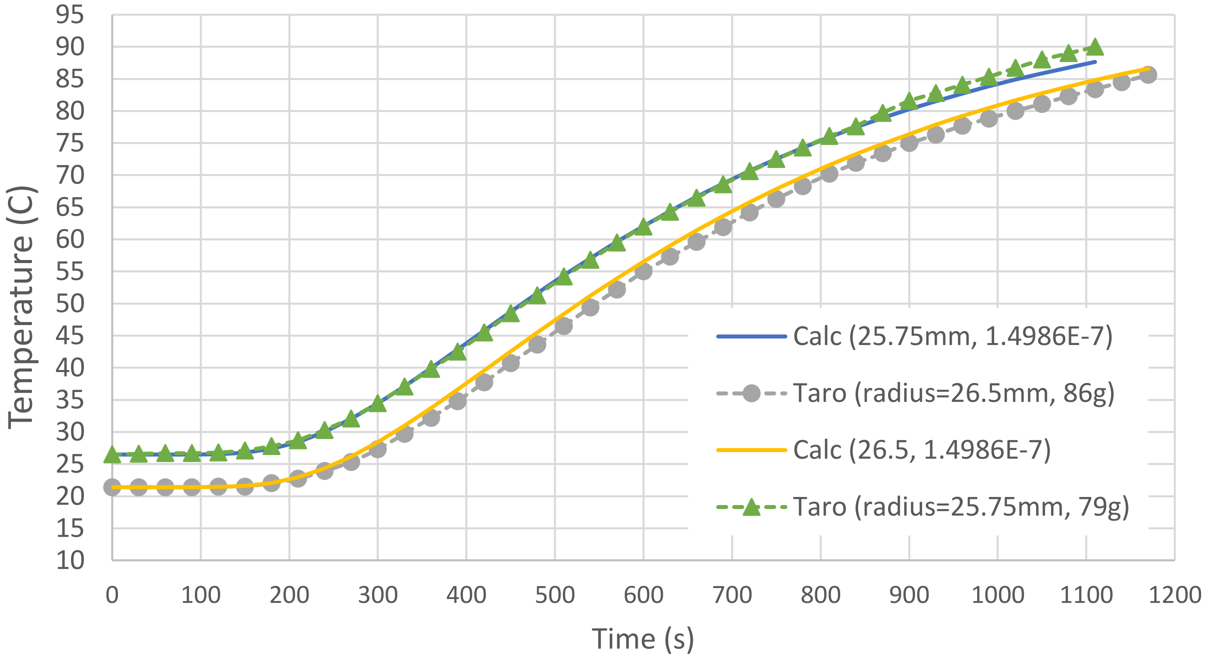


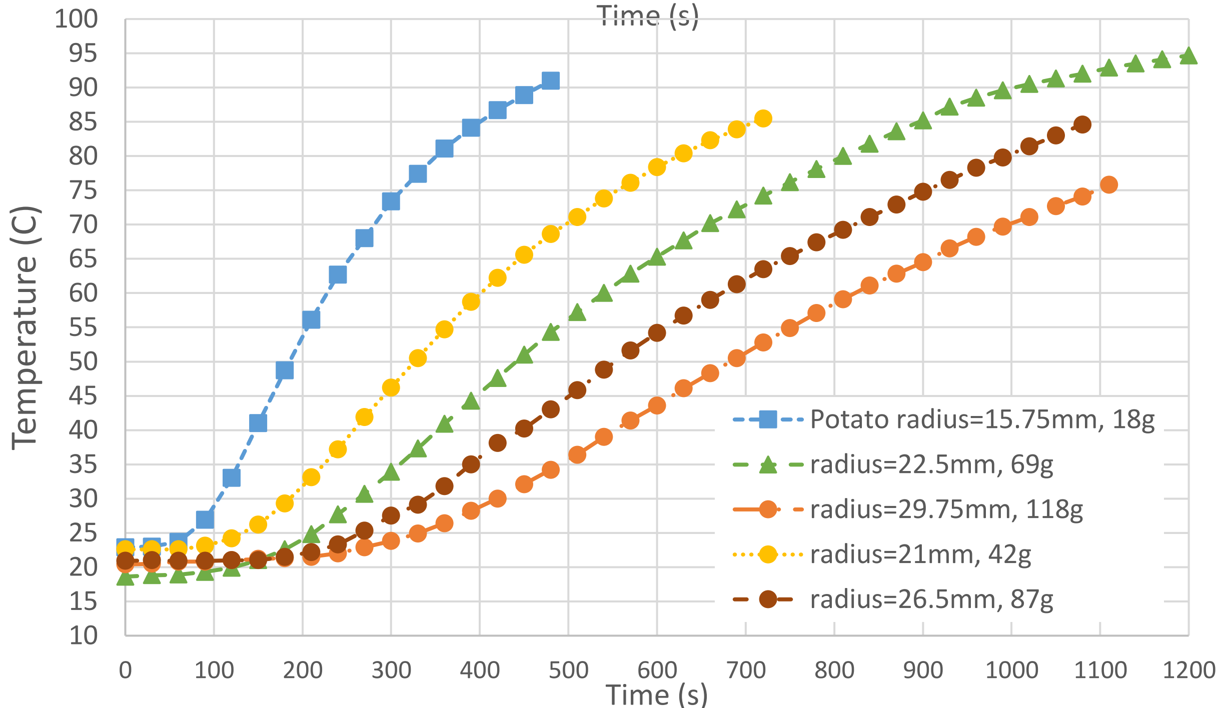

A potato sample with a diameter of 22.5mm was measured. The temperature of the center of the sphere sample is recorded as the function of time as shown in dots. The solid curves are calculated results based on the thermal diffusivity data as indicated in the figure.

# Experimental Measurement and Theoretical Simulation (fitting)

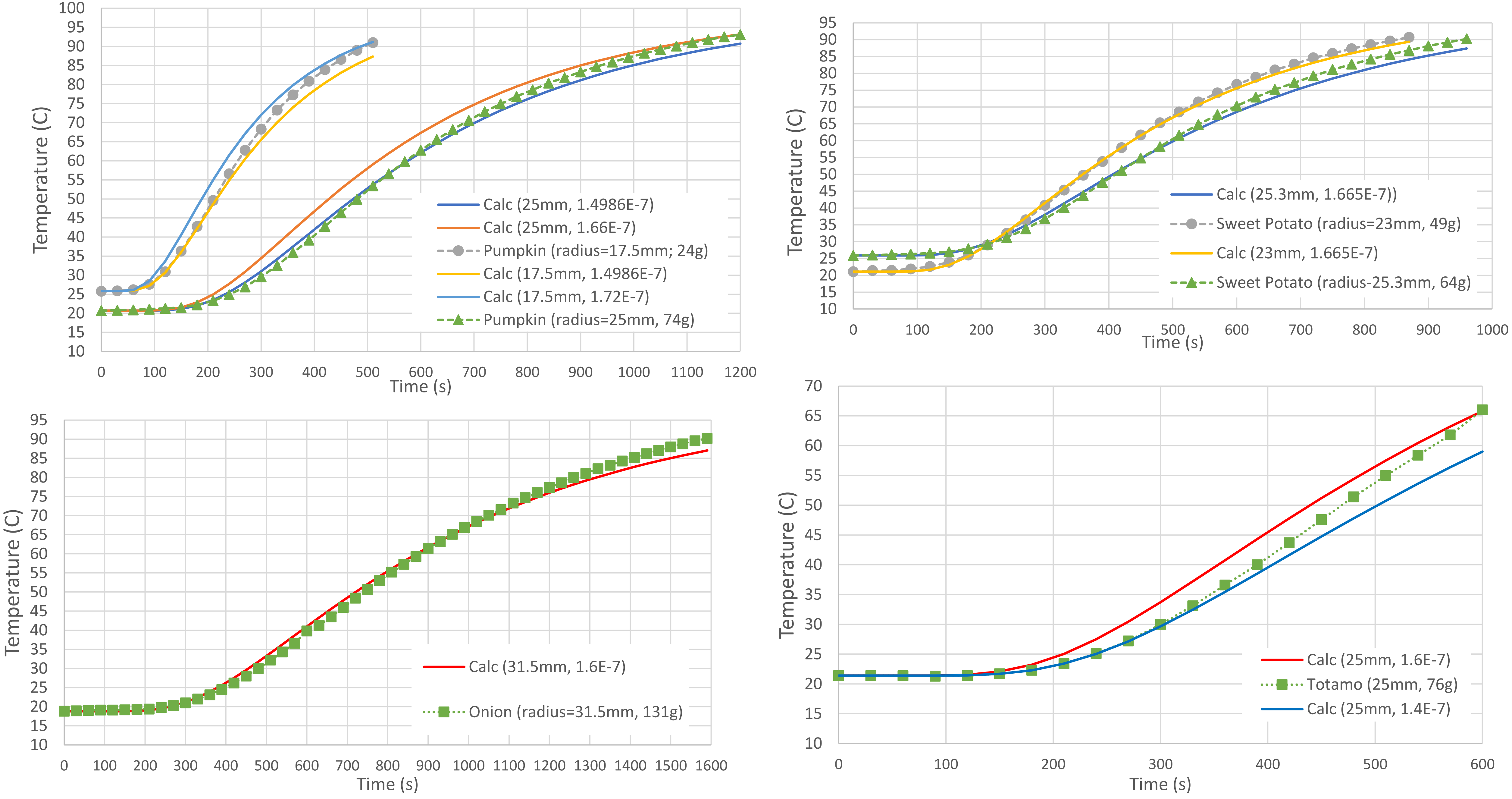


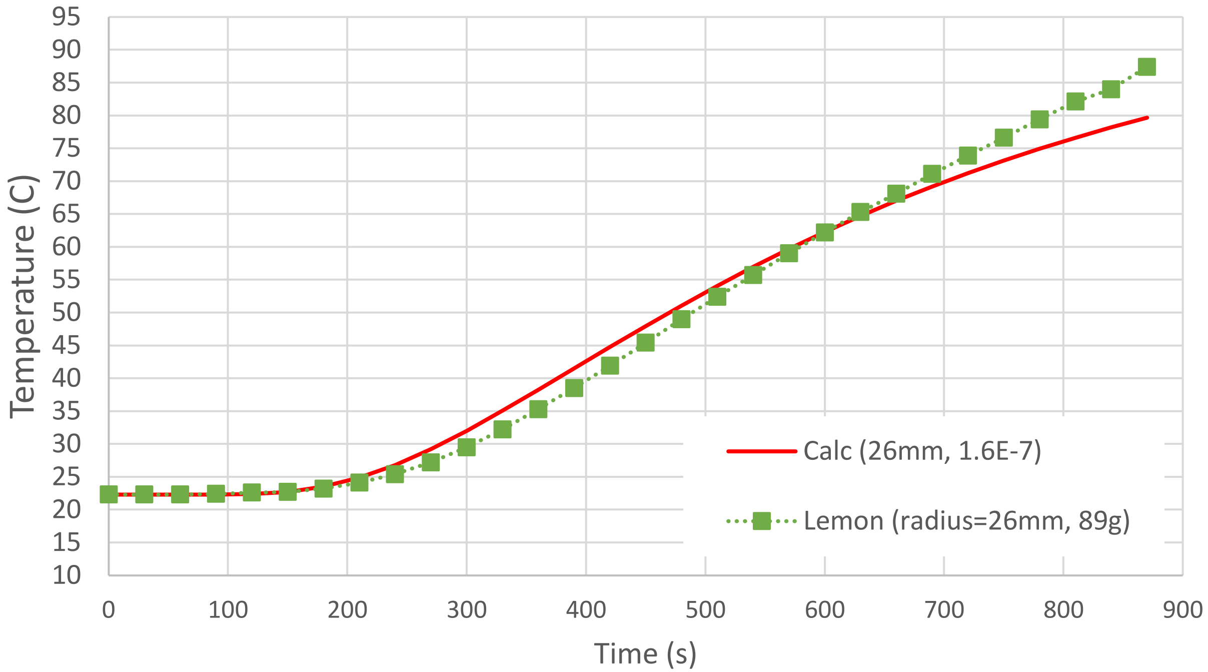



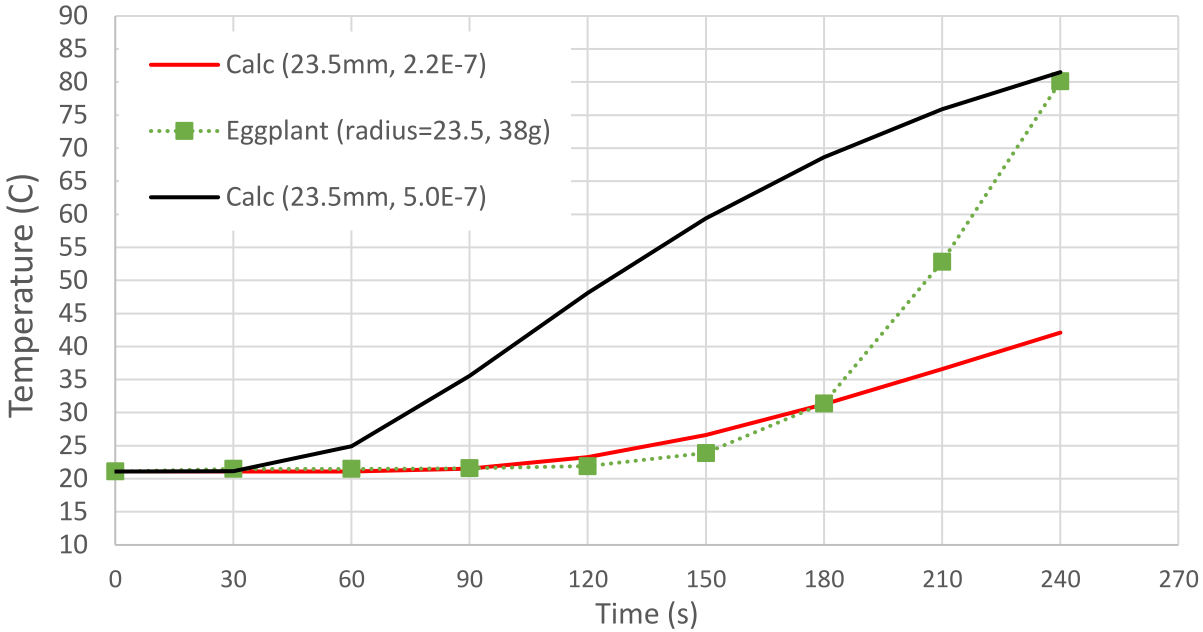



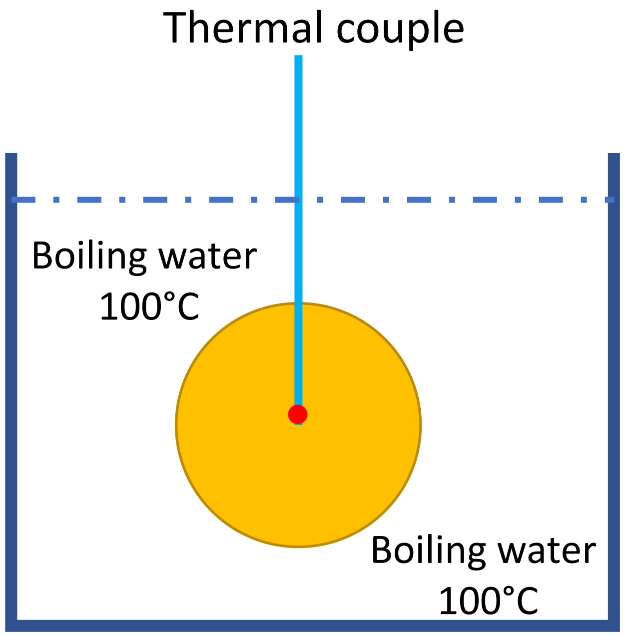
# Experimental Measurement and Theoretical Simulation (fitting)



# Experimental Measurement and Theoretical Simulation (fitting)




# Model of Physics

- Spherical approximation
- Heating uniformly from all directions
- Heating water temperature uniform
- The sample has an initial uniform temperature
- The sample is a uniform material with known thermal and physical parameters (i.e., diameter)
- The temperature at the center of the sphere is calculated (and measured) as the function of time.
- The diameter of the sample is a controlled variable
- Various samples are compared

# Heat Transfer Equation


Fourier's Law states that the heat flux q (in W/m²) is proportional to the temperature gradient, i.e., $q = -k\cdot\frac{dT}{dx}$ for one-dimensional systems. For the 3-dimensional system, $\vec{q} = -k\cdot\nabla T$ where $\vec{q}$ is a vector and $\nabla$ is the gradient. k is thermal conductivity in W/(cm· K).

$$Q_{net} = A\cdot(q_{x+\delta x} - q_x) = -kA\cdot\left[\frac{\partial T}{\partial x}_{x+\delta x} - \frac{\partial T}{\partial x}_{x}\right] = -kA\cdot\left[\frac{\frac{\partial T}{\partial x}_{x+\delta x} - \frac{\partial T}{\partial x}_{x}}{dx}\right]\cdot dx = -kA\cdot\frac{\partial^2 T}{\partial x^2}\cdot dx$$

$$-Q_{net} = \frac{dU}{dt} = \rho c A\cdot\frac{d(T - T_{ref})}{dt}\cdot dx = \rho c A\cdot\frac{dT}{dt}\cdot dx$$

This leads to the one-dimensional heat diffusion equation:

$$\frac{\partial^2 T}{\partial x^2} = \frac{\rho c}{k}\frac{\partial T}{\partial t} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Where T = T (x, t) and $\alpha = k/\rho c$ is the thermal diffusivity in m²/s, where ρ is the density (kg/m³) and c is the specific heat (J/(kg · K)).

In three-dimension, the heat transfer equation becomes:

$$\nabla\cdot\nabla T = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Where,

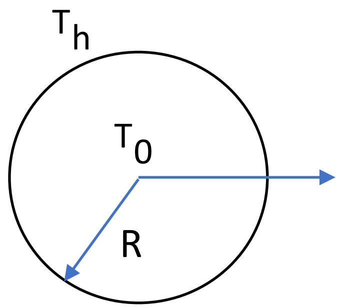
$$\nabla\cdot\nabla T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \quad (for\ Cartisian\ coordinates)$$

$$= \frac{1}{r^2\sin\theta}\left[\sin\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2 T}{\partial\varphi^2}\right] \quad (for\ spherical\ coordinates)$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 T}{\partial\theta^2}\right) + \frac{\partial^2 T}{\partial z^2} \quad (for\ cylinderical\ coordinates)$$

* Yifei "Jenny" Jin, Lisa R. Wang, and Jian Jim Wang, Physics in turkey cooking: Revisit the Panofsky formula, AIP Advances 11, 115316 (2021); https://doi.org/10.1063/5.0067811

# Heat Transfer for A sphere with azimuthal symmetry




For a sphere with azimuthal symmetry, during the heat transfer, we have $\frac{\partial T}{\partial \theta} = 0$ and $\frac{\partial^2 T}{\partial \varphi^2} = 0$, the heat transfer equation becomes

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right] = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Applying $V = r \cdot T$ to the above equation, for $0 \leq r \leq R$ we get:

$$\frac{\partial^2 V}{\partial r^2} = \frac{1}{\alpha}\frac{\partial V}{\partial t}$$

We can decouple V(r, t) into:

$$V(r,t) = R(r) \cdot T(t)$$

And we get:

$$\frac{\partial V}{\partial t} = R(r) \cdot \frac{\partial T}{\partial t} = R(r) \cdot T'(t)$$

And:

$$\frac{\partial^2 V}{\partial r^2} = T(t) \cdot R''(r)$$

Then, we have:

$$T(t) \cdot R''(t) = \frac{1}{\alpha} \cdot R(r) \cdot T'(t)$$

It can be rearranged into:

$$\frac{R''(r)}{R(r)} = \frac{1}{\alpha} \cdot \frac{T'(t)}{T(t)}$$

Since the left side is only be r-dependent and the right side is only be t-dependent, and since they equal to each other, they must be neither r- or t- dependent. So, we have:

$$\frac{R''(r)}{R(r)} = \frac{1}{\alpha} \cdot \frac{T'(t)}{T(t)} = -\lambda$$

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Then, we have:

$$R'' + \lambda R = 0$$

And

$$T' + \lambda\alpha T = 0$$

From the above equation, we have:

$$\frac{dT}{dt} = -\lambda\alpha T$$

$$\frac{dT}{T} = -\lambda\alpha \cdot dt$$

$$\int_0^t \frac{dT}{T} = -\lambda\alpha \cdot \int_0^t dt$$

$$lnT(t) - lnT(0) = -\lambda\alpha t$$

$$T(t) = e^{-\lambda\alpha t} \cdot T(0)$$

For $R'' + \lambda R = 0$

$$\frac{d^2R(r)}{dr^2} = -\lambda \cdot R(r)$$

$$R(r) = \mathrm{A}\cos\sqrt{\lambda}\cdot r + \mathrm{B}\sin\sqrt{\lambda}\cdot r$$

Now, we have:

$$V(r,t) = \sum_{\lambda}\left[\left(\mathrm{A}\cos\sqrt{\lambda}\cdot r + \mathrm{B}\sin\sqrt{\lambda}\cdot r\right)\cdot e^{-\lambda\alpha t}\right]$$

$$T(r,t) = \sum_{\lambda}\left[\left(\mathrm{A}\cos\sqrt{\lambda}\cdot r + \mathrm{B}\sin\sqrt{\lambda}\cdot r\right)\cdot \frac{e^{-\lambda\alpha t}}{r}\right]$$

For cooking (heating) food with food starting with low temperature $T_0$ and surrounded at high temperature (bath temperature) $T_h$, we have the following boundary conditions:

$T(r,o) = T_0$ $(0 \le r \le R)$, where R is the radius of the sphere.

$$T(\ge R,t) = T_h$$

We have:

$A = 0, and\ \lambda = \left(\frac{n\pi}{R}\right)^2$ where $n = 1, 2, 3, \ldots$

We then have:

$$T(r,t) = T_h - \frac{2R(T_h - T_0)}{\pi\cdot r}\sum_{n=1}^{\infty}\left[\frac{(-1)^{n+1}}{n}\sin\frac{n\pi r}{R}\cdot e^{-\alpha n^2\pi^2 t/R^2}\right]$$

for $(0 \le r \le R)$

We define

$\tau = \frac{R^2}{\pi^2\cdot\alpha}$ as the time constant.

Thus, we have:

$$T(r,t) = T_h - \frac{2R(T_h - T_0)}{\pi\cdot r}\sum_{n=1}^{\infty}\left[\frac{(-1)^{n+1}}{n}\sin\frac{n\pi r}{R}\cdot e^{-t/\tau}\right]$$

The temperature at the center of the sphere is (r = 0):

$$T_c = T_h - 2(T_h - T_0)\sum_{n=1}^{\infty}\left[(-1)^{n+1}\cdot e^{-t/\tau}\right]$$

We can spell out the equation with some of the initial (and deciding) terms:

$$T_c = T_h - 2(T_h - T_0)\{e^{-t/\tau} - e^{-4t/\tau} + e^{-9t/\tau} - e^{-16t/\tau} + e^{-25t/\tau} - e^{-36t/\tau} + e^{-49t/\tau} - \ldots\} \quad (1)$$

Where: $\tau = \frac{R^2}{\pi^2\cdot\alpha}$ and $\alpha = \frac{\mathrm{k}}{\rho c}$

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9 different types of foods along with their determined thermal diffusivities. The data were determined based on fitting the measured temperature curve with the theoretical model with the thermal diffusivity as the fitting parameter.

| | | Thermal diffusivity ($10^{-7}$ $m^2/s$) | |
|---|---|---|---|
| Food | Diameter (mm) | Low end value | High end value |
| Potato | 45.4 | 1.32 | 1.42 |
| Potato | 45 | 1.32 | 1.42 |
| Potato | 50 | 1.32 | 1.42 |
| Potato | 40 | 1.32 | 1.42 |
| Potato | 31.5 | 1.32 | 1.48 |
| Potato | 59.5 | 1.32 | 1.50 |
| Potato | 42 | 1.32 | 1.48 |
| Potato | 46 | 1.32 | 1.50 |
| Potato | 53 | 1.32 | 1.50 |
| Potato (reheated) | 51 | 1.52 | 1.60 |
| Pumpkin | 50 | 1.50 | 1.66 |
| Pumpkin | 35 | 1.50 | 1.72 |
| Sweet potato | 50.6 | 1.66 | 1.84 |
| Sweet potato | 46 | 1.66 | 1.75 |
| Taro | 51.5 | 1.50 | 1.60 |
| Taro | 53 | 1.40 | 1.50 |
| Radish | 40 | 1.30 | 1.40 |
| Radish | 41 | 1.55 | 1.65 |
| Onion | 63 | 1.60 | 1.78 |
| Eggplant | 47 | 2.20 | 5.00 |
| Lemon | 52 | 1.50 | 1.70 |
| Tomato | 50 | 1.40 | 1.60 |

## Comparison with Data Founded in the Literature

- T.R.A. Magee et. al. measured the thermal diffusivity of potato using a thermal diffusivity tube under transient heat transfer conditions by two different methods, the log method and the slope method, both based on the solutions of the Fourier equation. Both methods gave similar results for potato, 1.30 × 10-7 m2/s and 1.44 × 10-7 m2/s.
- M. A. Rao et. al. reported the average thermal diffusivity value for potato as 1.70 x 10-7 m2/s.
- According to the Engineering ToolBox, the thermal diffusivity for potato is 1.23x10-7 m2/s for cooked and mashed potato and 1.70x10-7 m2/s for flesh potato. For tomato pulp, the thermal diffusivity is 1.48x10-7 m2/s.
- A. Farinu et. al. determined the thermal diffusivity for sweet potato to be 1.2 × 10-7 m2/s.
- Obot et. al. measured the thermal diffusivity for white radish to be 1.869×10-7 m2/s.
- In Encyclopedia of Food Sciences and Nutrition, potato has a reported thermal diffusivity of 1.3x10-7 m2/s.
- H. Kocabiyik et. al. measured the thermal diffusivity of pumpkin seeds to be 1.289 × 10-7 m2/s.
- A.E. Drusas et. al. measured tomato paste, the thermal diffusivity of tomato pastes was estimated as 1·42 × 10-7 m2/s.
- Abhayawick et. al. measured onion for a value between 1.1x10-7 m2/s and 1.5x10-7 m2/s depending on the moisture's content.
- Luis A. Minim et. al. measured lemon Juice and had a result between 1.160x10-7 and 1.785x10-7 m2/s.
- We have repeated the measurements for potato with multiple samples. The results we obtained are very consistent, which indicates the consistency of this measurement method. The obtained result for potato is between 1.32x10-7 m2/s and 1.50x10-7 m2/s with a tight range, and the result is in excellent agreement with some of the reported values, and in good agreement with all the other reported values.

# Discussions

Any potential measurement errors could come from the following factors.

(1) The thermo couple temperature measurement error mainly comes from the position accuracy. We need to measure the center temperature of the spherical sample. However, this error is believed to be small based on the excellent repeatability and agreement between samples of the same type and samples with different radii.
(2) The thermo couple sensor tip might move during the heating process. Such problem might cause some temperature data irregularity for a later part of the heating curve, as indicated in one of the experimental figures (after 700 seconds in time).
(3) The error of the measurement of the sample’s diameter (radius). This error could be reduced with the help of the accurate mass measurement.
(4) The error of the shape deviation from the perfect sphere. To analyze the impact of the shape deviation, we define a shape factor S. Since the transferred heat is proportional to the surface area and the received energy per volume is inversely proportional to the total volume, thus the inverse of the baking time (or heating time) is proportional to the surface area and inversely proportional to the volume of the piece. Our analysis indicates that small shape deviation leads to very small impact to the accuracy of the final data, as indicated by the good agreement between different samples with random deviation of the shape.