Physics behind Cooking Intelligence

Lisa (Nico) Wang

The Pennington School

112 W Delaware Ave, Pennington, NJ 08534

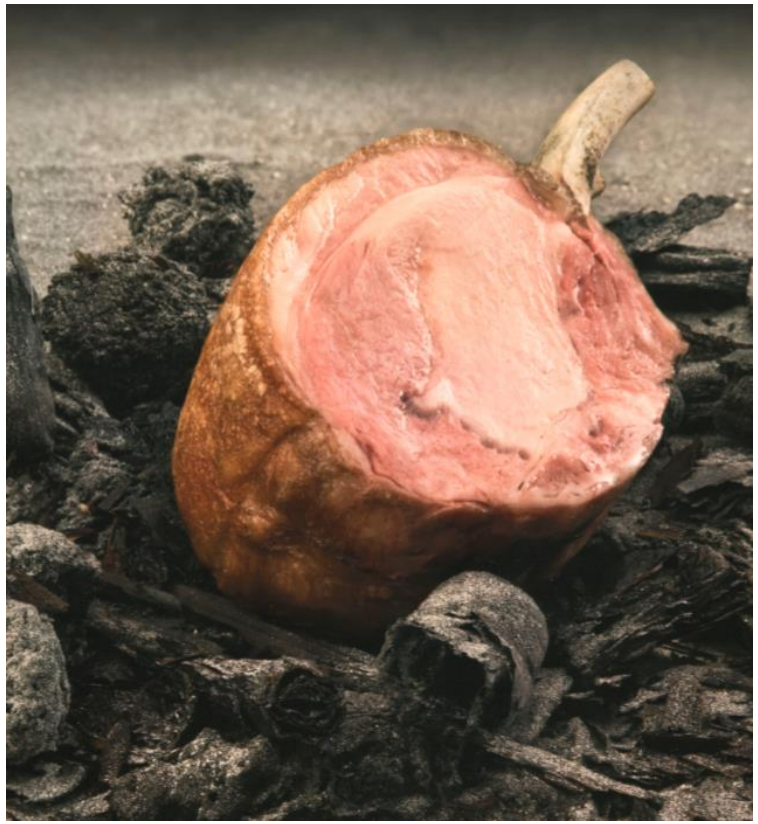
Cooking Contains Rich Science/Physics!

- ✓ Cooking provides such a rich set of opportunities and examples to learn and study science and in particular, physics.
- ✓ Trying to understand the science in cooking, paired with edible lab experiments, generate enthusiasm and provide strong motivation for people to learn physics.
- ✓ Physical understanding of the cooking process of food is vital to understand the underlying physical phenomena and to help optimize the culinary quality in terms of process, texture, and flavor optimization, and as well as safety of meat and other types of foods' consumption.
- ✓ A great example of applying physical thinking to the culinary arts is the story about how SLAC former director, Professor WKH 'Pief' Panofsky, developed its formula for baking of turkeys.

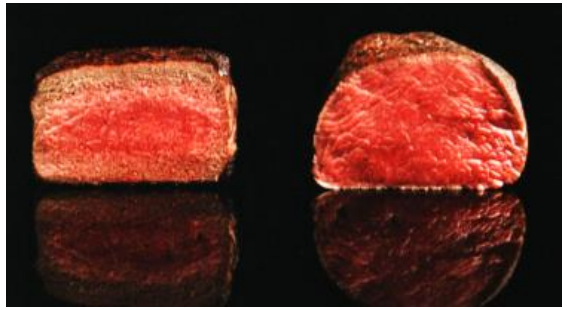
1. Nathan Myhrvold, Chris Young, and Maxime Bilet, Modernist Cuisine (The Art and Science of Cooking), Volume 1, ISBN: 978-0-9827610-0-7, First edition, 2011.
2. Jeff Potter, Cooking for Geeks: Real Science, Great Hacks, and Good Food, ISBN-13: 978-1491928059, August 17, 2010.
3. Joseph J. Provost, Keri L. Colabroy, Brenda S. Kelly, Mark A. Wallert, The Science of Cooking: Understanding the Biology and Chemistry Behind Food and Cooking, ISBN-13: 978-1118674208, 1st Edition, 2016.
4. Eileen Yin-Fei Lo, Mastering the Art of Chinese Cooking, October 28, 2009.

The Long and Rich History of Smart Cooking

- Chinese has been cooking typical and famous Chinese dishes like "Fish flavored shredded thread-pork", "Pepper shredded beef", "Kung Pao chicken", "Sichuan boiled fish", "Braised pork balls", "General Tso's chicken", "Chinese fried rice", "Sweet & sour pork", "Mapo tofu", "Chow Mein", "Shredded potato", or essentially similar dishes for hundreds if not thousands of years.
- The stir-frying cooking technique is one of the major cooking methods in Chinese or Asian (Indian) culinary. Stir-frying originated during the Han Dynasty (206BC – 220AD). Archeologists found evidences of woks and thinly sliced food in ancient civilization sites. Stir-frying became the dominant and primary Chinese cooking method during Ming Dynasty (1368 – 1644).
- The ancient nomadic lifestyle in China required the people then to be able to cook fast, clean easily, carry effortless, use minimal cooking oil, and consume minimal precious fuel which means the cooking method had to be most energy efficient. The wok with a close to parabola shape can be heated up fast with least energy and concentrate the heat to the food at the bottom of the wok from the perspectives of conduction, convection, and radiation. The high heat nature and less cooking oil required accidentally led to more healthy food. The method eventually spread quickly to Japan around 1868 to 1912 and then to north America and the rest of the world in the 20th century. The chronic shortage of fuel, i.e., wood, coal, and other fuel types, might be one of the major reasons behind stir-frying's popular acceptance in ancient and modern China and other parts of the world.
- Another key leading to the feasibility of fast-speed cooking or shortest cooking time with stir-frying belongs actually to the main topic of this research. Before cooking, stir-frying requires the raw foods, no matter in what kind of original sizes and shapes and materials, to be shredded into small pieces in the shapes of thread/wire, sphere, thin slice, cube, etc.



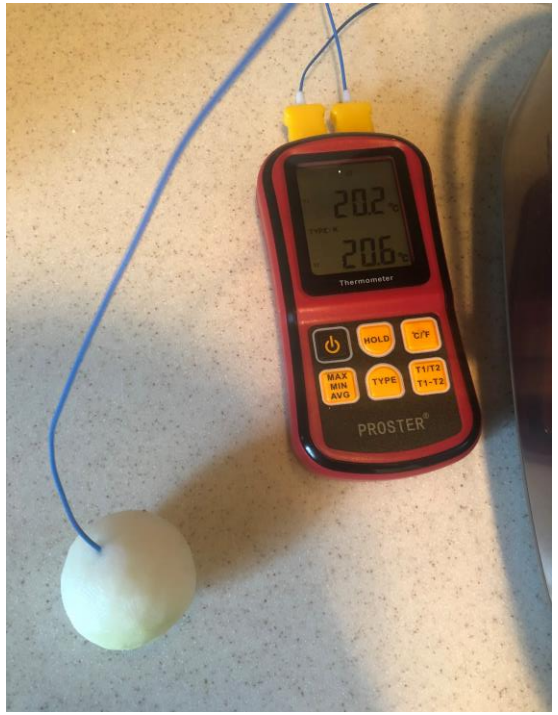




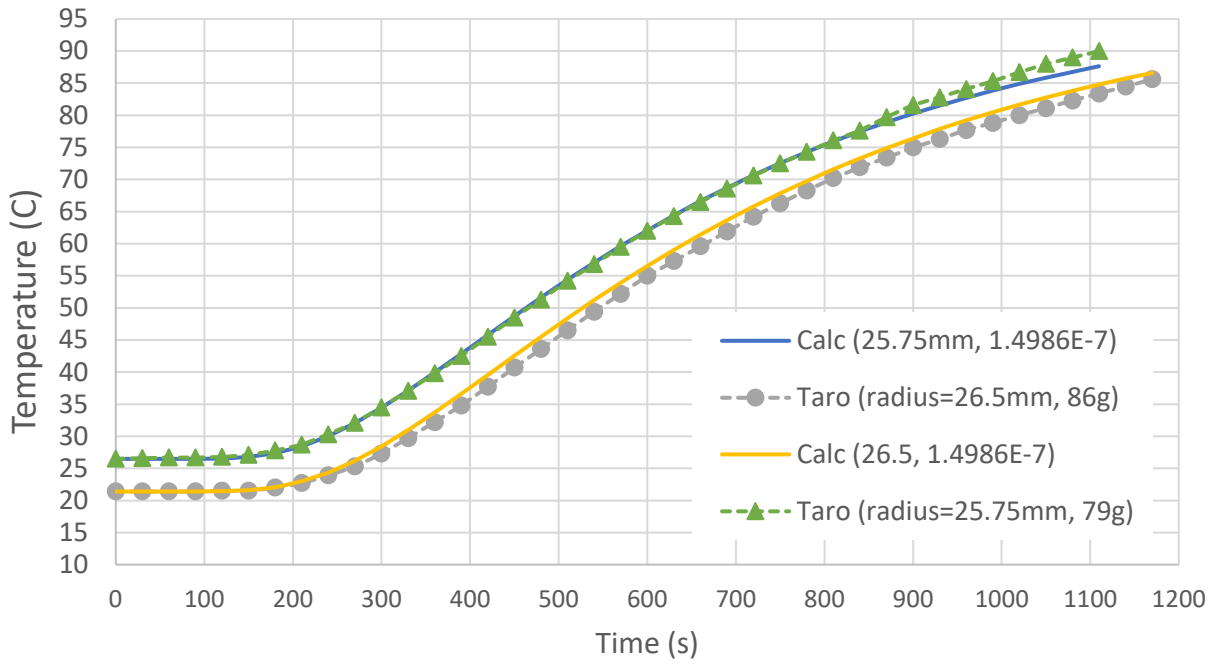


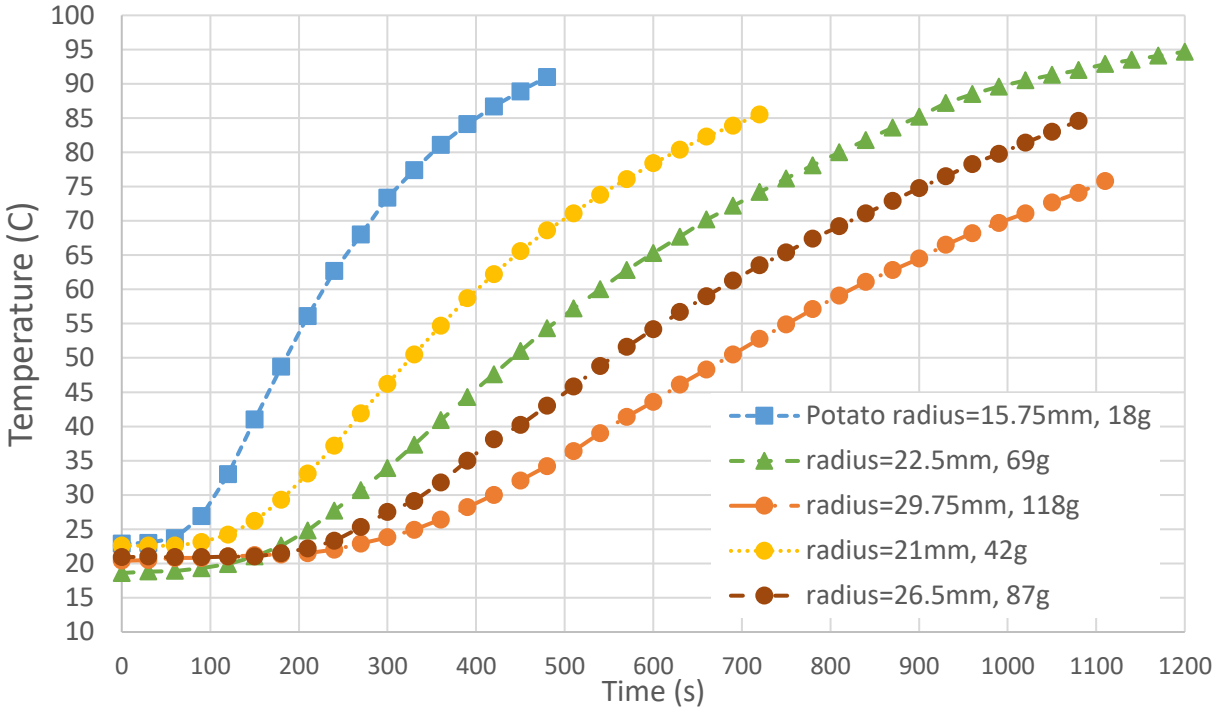

A Simple Method to Determine Thermal Diffusivity

- ❑ Thermal couples with small diameters are used to measure the center temperature of the samples.
- ❑ The samples are boiled in boiling water (100 C).
- ❑ Track the center temperature rise as the function of time.



Experimental Measurement and Theoretical Simulation (fitting)





Taro samples with two different diameters (25.75mm and 26.5mm) were measured. The temperature of the center of the sphere sample is recorded as the function of time as shown in dots (26.5mm sample) and in triangles (25.75mm sample). The solid curves are calculated results based on the thermal diffusivity data as indicated in the figure.

Potato samples with 5 different diameters (15.75mm, 21mm, 22.5mm, 26.5mm, and 29.75mm) were measured. The temperature of the center of the sphere sample is recorded as the function of time as shown in dots.

9 different types of foods along with their determined thermal diffusivities. The data were determined based on fitting the measured temperature curve with the theoretical model with the thermal diffusivity as the fitting parameter.

		Thermal diffusivity (10^{-7} m^2/s)	
Food	Diameter (mm)	Low end value	High end value
Potato	45.4	1.32	1.42
Potato	45	1.32	1.42
Potato	50	1.32	1.42
Potato	40	1.32	1.42
Potato	31.5	1.32	1.48
Potato	59.5	1.32	1.50
Potato	42	1.32	1.48
Potato	46	1.32	1.50
Potato	53	1.32	1.50
Potato (reheated)	51	1.52	1.60
Pumpkin	50	1.50	1.66
Pumpkin	35	1.50	1.72
Sweet potato	50.6	1.66	1.84
Sweet potato	46	1.66	1.75
Taro	51.5	1.50	1.60
Taro	53	1.40	1.50
Radish	40	1.30	1.40
Radish	41	1.55	1.65
Onion	63	1.60	1.78
Eggplant	47	2.20	5.00
Lemon	52	1.50	1.70
Tomato	50	1.40	1.60

A simple, low-cost, fast, and accurate method to measure the diffusion coefficient of salt in various foods

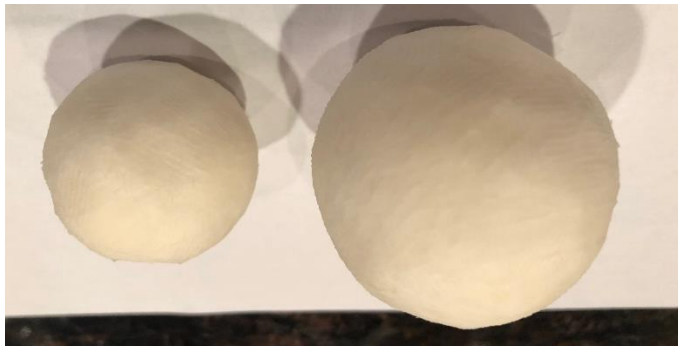
- ✓ Simple: samples are very easy to prepare.

- ✓ Low-cost: the total cost for the materials, tools, and instruments used in this research is less than $900!
    - The compact salt meter (LAQUAtwin-salt-11) made by Horiba: $180.
    - 200g x 0.1mg Digital Analytical Balance Lab Precision Scale from U.S. Solid: $480.
    - Caliper: $20.
    - All the food materials: $50.
    - Salt: $30.
    - Other containers and cooking wares: $100.

- ✓ Fast: it takes less than 20 minutes to measure each sample (excluding the brine times).

- ✓ Accurate: the measured results are consistent and accurate.

Sample Preparation

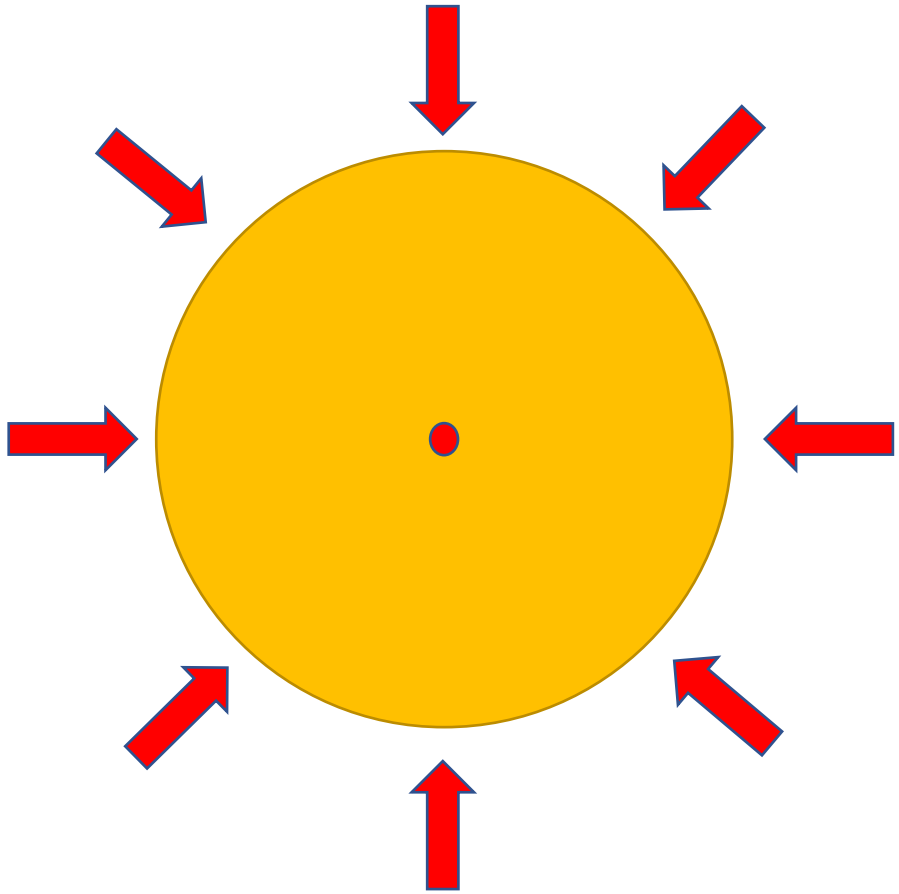


Different foods are cut into nearly perfect spheres with different diameters (top: sweet potato, middle: radish, right: taro).

Sample Preparation

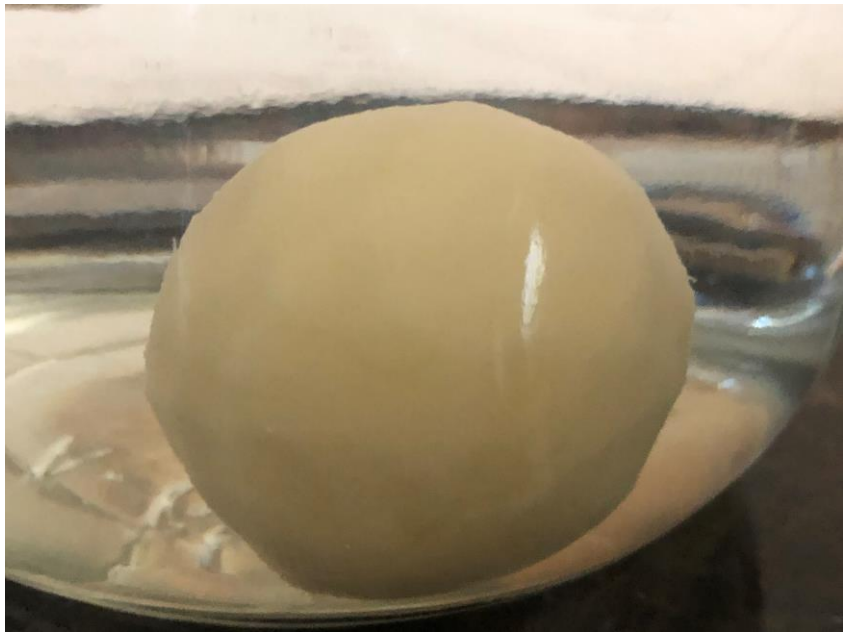
Different foods are cut into nearly perfect spheres with different diameters (top: sweet potato, middle: radish, right: taro).

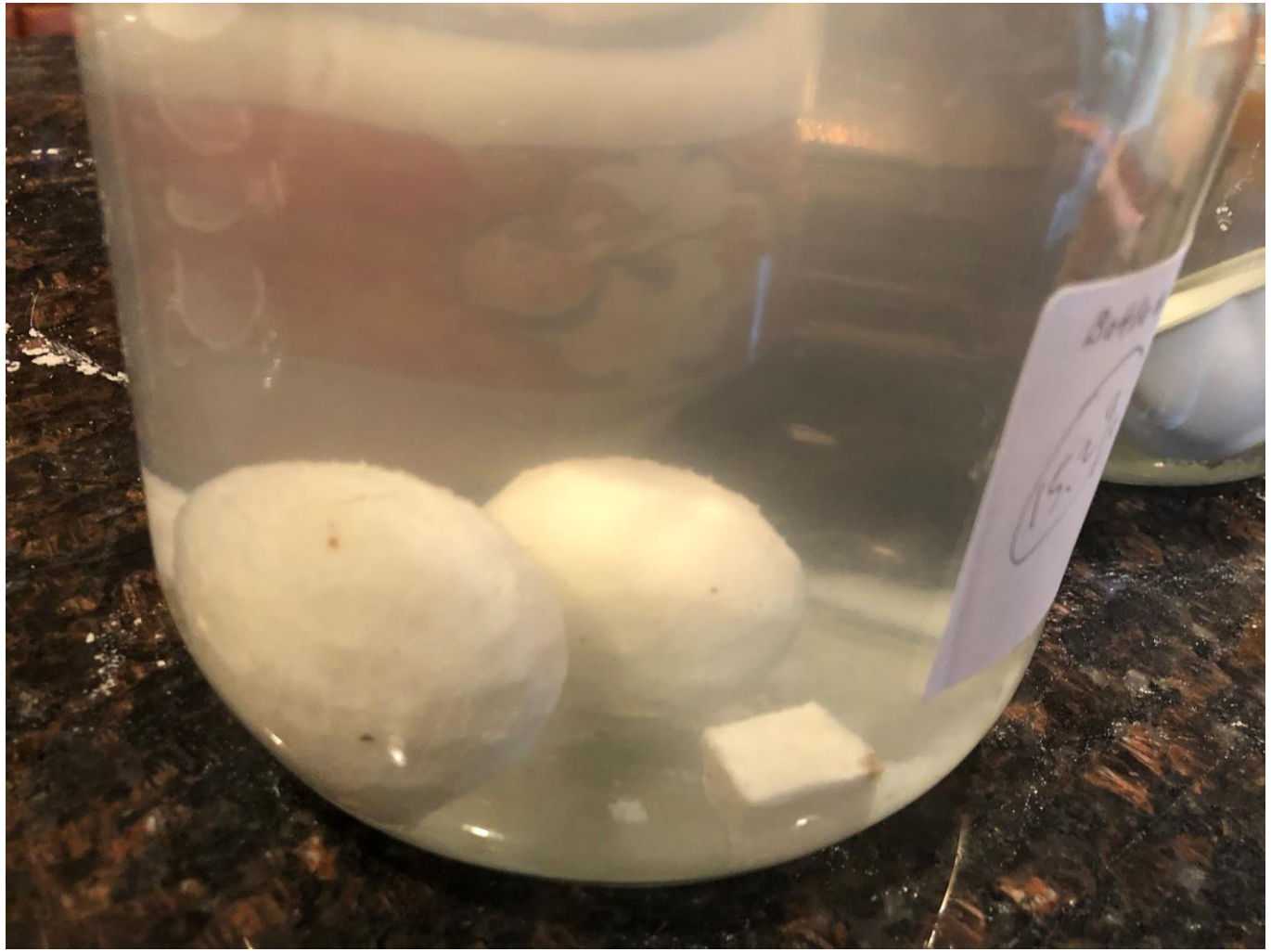
Why Spherical Shape?



- ❑ Spherical symmetry makes the distance from the center only “parameter”.
- ❑ Theoretically, it is easy to simulate.
- ❑ Experimentally, it is easy to measure.
- ❑ The comparison between the theoretical calculation and the experimental measurement become possible and straightforward.

Experiment in Process: Brine





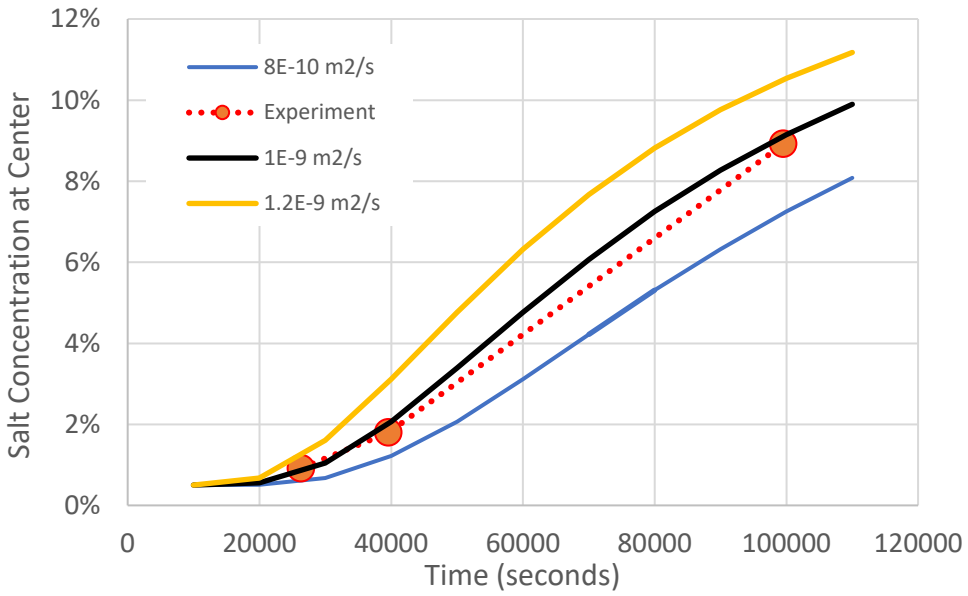
Brine durations: 1 hour to 24 hours

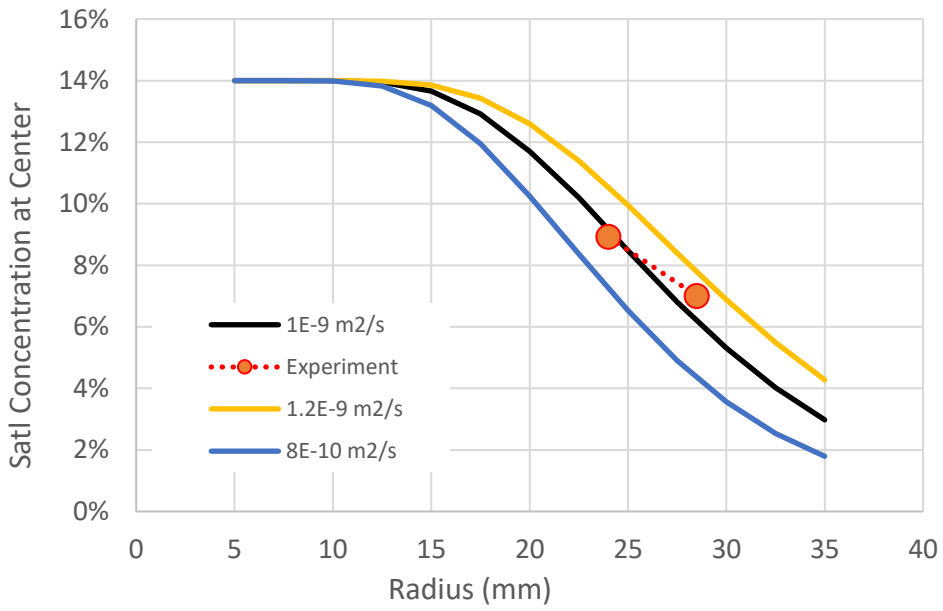
Salt Concentration Measurement Method

- A compact salt meter (LAQUAtwin-salt-11) made by Horiba was used to determine the salt concentration. HORIBA's unique compact meter integrates the electrode, display and sample container to enable simple, effective on-site testing by direct measurement from a single drop. The LAQUAtwin-salt-11) can measure between 0% to 10% in absolute concentrations with a relative precision of +/-4%.

- The sample is cut and a small piece (about 1 mg) is taken from the center of the sample and then is measured with the compact salt meter.

Experimental Measurement and Theoretical Simulation (fitting)

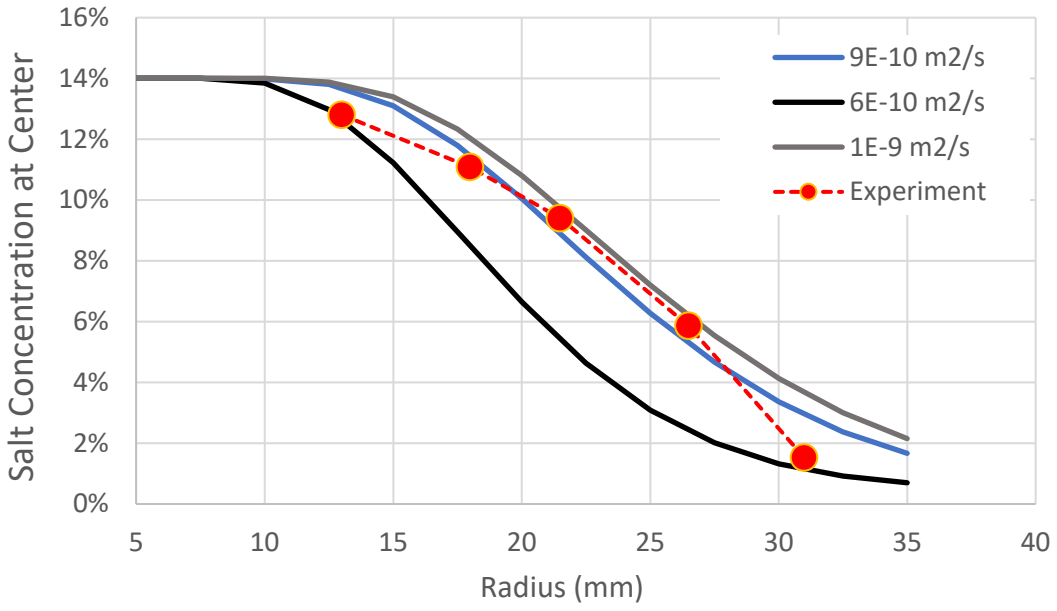




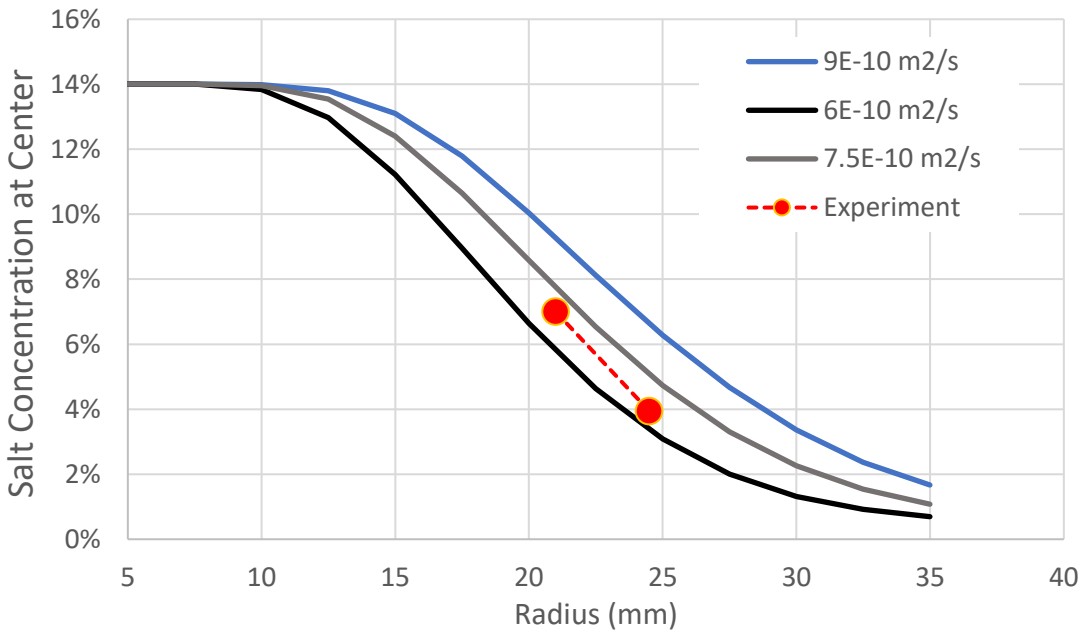
Potato samples with same radius (24mm) were measured. The salt concentration at the center of the sphere sample is recorded as the function of brine time as shown in dots. The solid curves are the simulation with the diffusion coefficient as the only variable.

Two potato samples with different radius (24mm and 28.5mm) were measured. The brine time duration is kept at 24 hours (86400 seconds). The salt concentration at the center of the sphere sample is recorded as the function of radius as shown in dots. The solid curves are the simulation with the diffusion coefficient as the only variable.

Experimental Measurement and Theoretical Simulation (fitting)

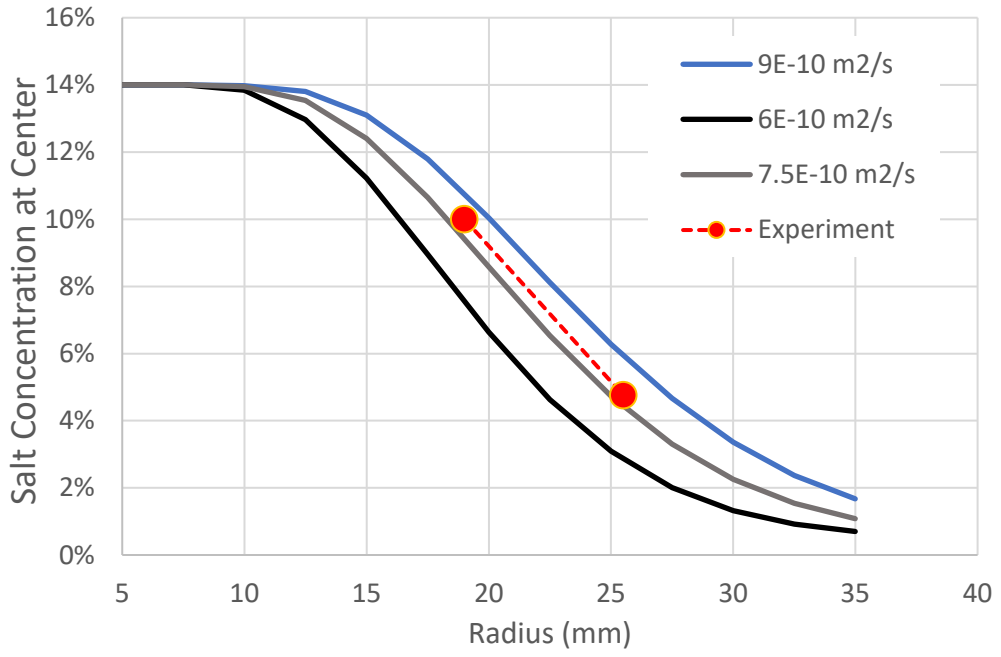


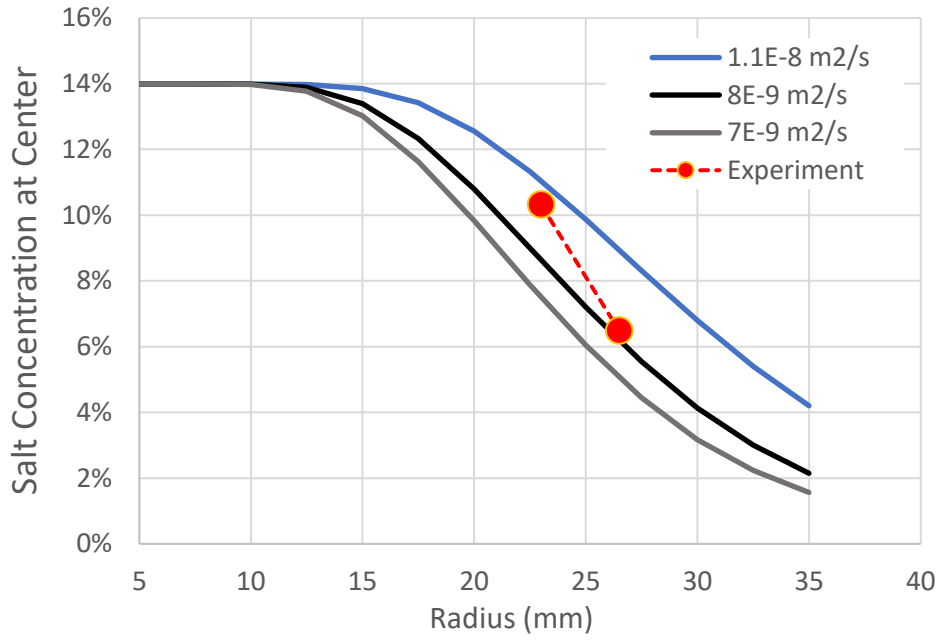
Five sweet potato samples with different radius (13mm, 18mm, 21.5mm, 27mm, and 31mm) were measured. The brine time duration is kept at 24 hours (86400 seconds). The salt concentration at the center of the sphere sample is recorded as the function of radius as shown in dots. The solid curves are the simulation with the diffusion coefficient as the only variable.



Two taro samples with different radius (21mm and 24.5mm) were measured. The brine time duration is kept at 24 hours (86400 seconds). The salt concentration at the center of the sphere sample is recorded as the function of radius as shown in dots. The solid curves are the simulation with the diffusion coefficient as the only variable.

Experimental Measurement and Theoretical Simulation (fitting)

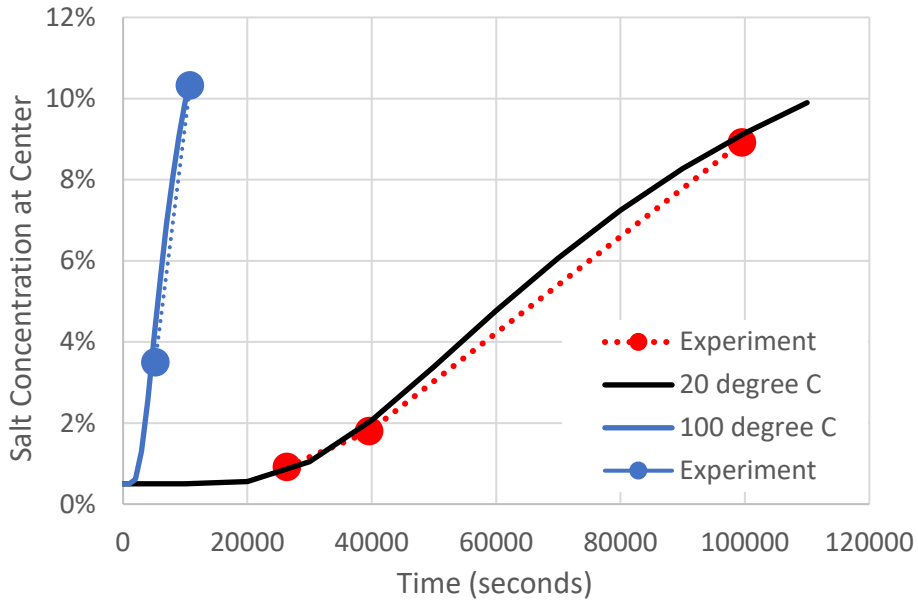




Two radish samples with different radius (19mm and 25.5mm) were measured. The brine time duration is kept at 24 hours (86400 seconds). The salt concentration at the center of the sphere sample is recorded as the function of radius as shown in dots. The solid curves are the simulation with the diffusion coefficient as the only variable.

Two potato samples with different radius (24mm and 26mm) were measured at 100℃. The brine time duration is kept at 3 hours (10800 seconds). The salt concentration at the center of the sphere sample is recorded as the function of radius as shown in dots. The solid curves are the simulation with the diffusion coefficient as the only variable.

Experimental Measurement and Theoretical Simulation (fitting)



Potato samples with same radius (24mm) were measured at two different temperatures (20°C and 100°C). The salt concentration at the center of the sphere sample is recorded as the function of brine time as shown in dots. The solid curves are the simulation with the diffusion coefficient as the only variable.

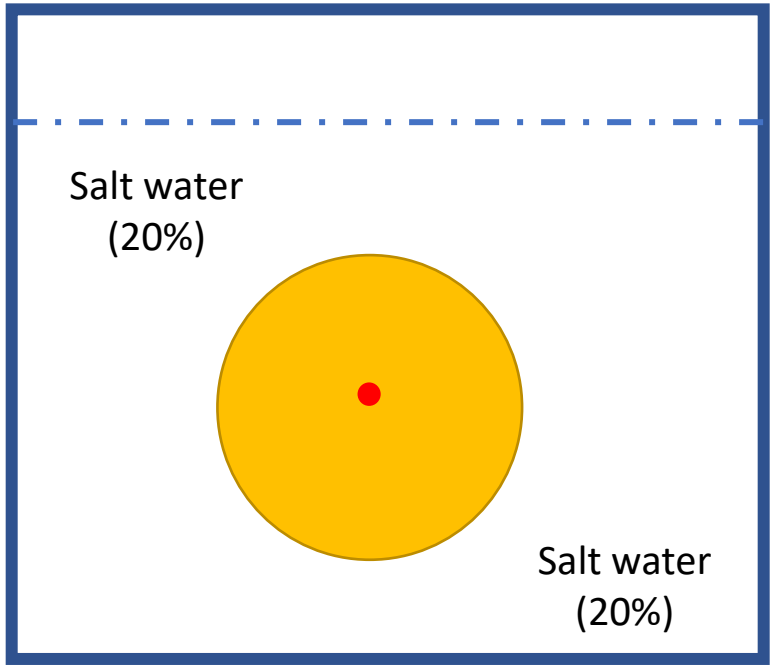
The dependency of the diffusion on temperature is described by the Arrhenius equation as follow:

$$D_e = D_0 e^{-\frac{E_a}{RT}}$$

Where D_e is the effective diffusion coefficient in m^2/s. E_a is the activation energy in meV or J/mol. D_0 is the pre-exponential factor in m^2/s. R is the gas constant (8.314 J/mol K). T is the absolute temperature.

With T = 373K for our case, and De is between 8×10^{-9} to 1.1×10^{-8} m2/s, and D_0 is between 8×10^{-10} to 1.2×10^{-9} m2/s, we derive the activation energy to be around 74meV or 7.13 kJ/mol.

Model of Physics



- Spherical approximation
- Diffuse uniformly from all directions
- Water salt concentration uniform
- The sample has an initial uniform zero (or close to zero) salt concentration
- The sample is a uniform material with physical parameters (i.e., diameter)
- The salt concentration at the center of the sphere is calculated (and measured) as a function of diffusion time.
- The diameter of the sample is a controlled variable
- Various samples are compared

Modeling of the Diffusion of Salt

The transportation of salt, sugar, water, oil, and other molecules and ions, is governed by the mass transfer process, which is described by an equation so-called Fick's 2nd Law similar to the heat transfer equation:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \mathrm{c} \;\; = \;\; \frac{1}{D}\frac{\partial C}{\partial t}$$

Where C = C (x, t) is the concentration of the molecules or ions, which is the function of location and time. D is the diffusion coefficient of the molecules or ions in m^2/s.

Five different types of foods along with their determined diffusion coefficient of salt. The data were determined based on fitting the measured salt concentration at the center of a spherical shaped food with the theoretical model with the diffusion coefficient as the fitting parameter.

	Diffusion Coefficient (10^{-10} m^2/s) @ 20℃	
Food	Low end value	High end value
Potato	8.0	12.0
Pumpkin	11.0	13.0
Sweet potato	6.0	10.0
Taro	6.0	7.5
Radish	7.5	9.0

Cooking Sciences and Their Fantastic Aspects

- **Heat and/versus Salt Diffusion: The Key to Understand Cooking**

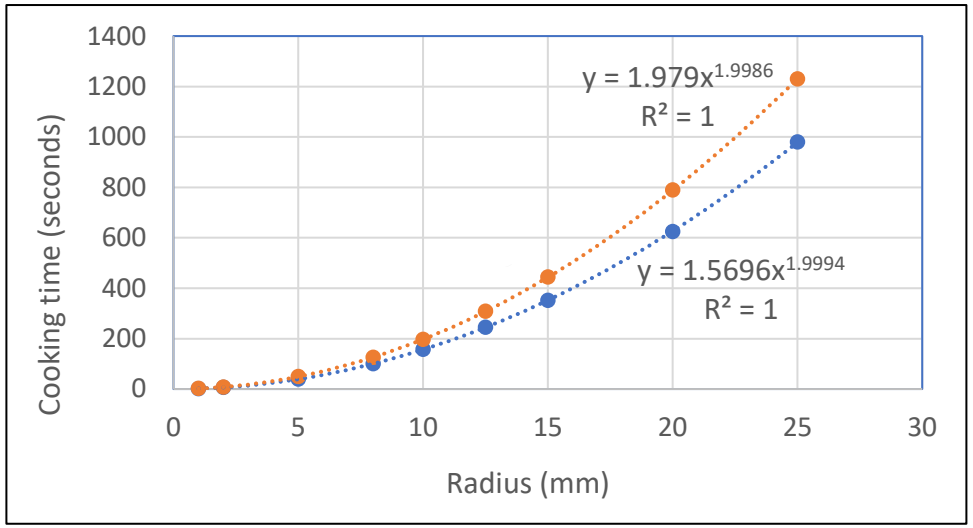
- Why can’t we cook the duck eggs while getting them salted to the way we like within the same process and duration? Cooking the eggs only takes about 5 to 10 mins, however, the salting process to make the famous salted duck eggs requires 20 or even 30 days. The similar question exists for salted pork, salted fish, salted duck, salted vegetables, and many more salted foods.
- For the popular French Fries, in a cooking oil bath with a temperature around 150-160 ℃, with a dimension of typical 5x5 mm in cross section, it only takes 30 seconds to get them cooked, i.e., to reach over 100 ℃ across the whole fries. Why do we need to have a double frying process in the standard McDonald’s recipe, with the first frying for 5 mins at 163 ℃, that alone is 10 times of the duration required to heat the fries from the heating perspective, and the second frying for 2-3 mins at even higher temperature like 180 ℃. The intervals between the two frying can be days or even months?
- Why different types of noodles or spaghettis with similar cross-section dimensions (diameters) would need a cooking time, while being soaked in boiling water, ranging from 1-3 mins to 12-14 mins, an order of magnitude in difference?
- While Chinese stir-frying dishes only take about 3 mins to cook, is it because of the need of heating process alone? If we look at the hotpot, the similar-sized food only takes about 20 seconds to dip in the 100 ℃ pot to be ready to eat (but with a dipping source to gain flavor). What prevents us from further shrinking down the cooking time in the stir-frying cooking?

The quadratic dependence of the diffusion to the dimension

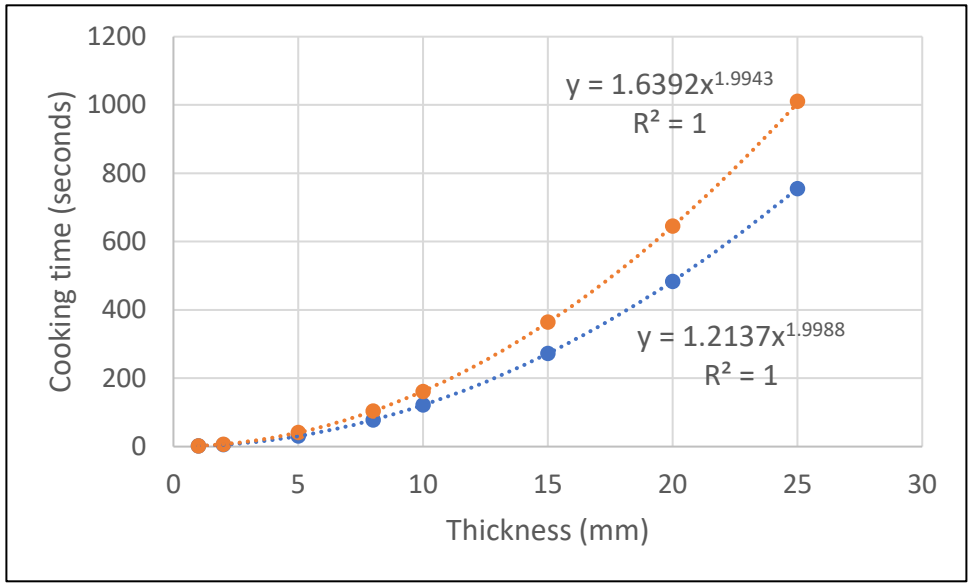
- ✓ It is clear that the cooking time to reach the same center temperature also has a perfect (with the fitting $R^2 = 1.00$) square relationship with the thickness of the meat slices. For a meat slice with a thickness of about 1 to 2 mm, it only takes about few seconds (1.21 to 6.5 second) to reach the required temperatures. For a meat slice of 25 mm in thickness (i.e., about 1 inch), it will take 755 second to reach 75 ℃ at the center of the slice, and takes 1010 second to reach 85 ℃ at the center of the slice.

- ✓ We see that with a similar critical dimension, it takes much longer for a slice to reach the same center temperature as what a ball does. A slice with thickness of 25 mm requires a cooking time which is equivalent to a ball with radius of about 23 mm or a diameter of about 46 mm which is 2 times of the thickness of the slice. For the slice, the heat goes in from one direction which is perpendicular to the slice surface. In comparison, for the ball, the heat goes in from all directions.

The Quadratic Dependence of Cooking Time to Food Critical Dimension



Cooking time of meatballs, in seconds, as function of the radius of the meatballs, in millimeters. The blue dots are the ones with the temperature at the center of the meatball reaching 75°C, and the orange dots are the ones with the center temperature reaching 85°C. The dotted curves are the fitting curves with the fitting parameters listed on the figure.



Cooking time of thin meat slices, in seconds, as function of the thickness of the meat slices, in millimeters. The blue dots are the ones with the temperature at the center of the meat slice reaching 75°C, and the orange dots are the ones with the center temperature reaching 85°C. The dotted curves are the fitting curves with the fitting parameters listed on the figure.

Explain Well about Cooking Times for Various Foods

- ✓ In the discussion above, a cube with a side length of a is equivalent to a sphere with diameter of 0.8a. That is, for a meatball with 25 mm (1 inch) in diameter, it takes the same time to reach the same center temperature for a meat cube with a size length of 20 mm. It is about the same if this applies to a short cylinder shape.

- ✓ Our calculated cooking times for hot-potting, boiling, stewing meats in the shape of balls, cubes, short cylinders, thin slices match very well with the corresponding times from our real time experiences.

- ✓ For instances, in Chinese hot-pot case, we typically only dip the meat (lamb, beef, fish) slices (with thickness from 0.5 to 2 mm) for only a few seconds. Our calculations call for 1.5 seconds for 1 mm thick meat slice and about 5-7 seconds for 2 mm thick meat slices. For fish or beef balls, the typical diameters are around 20 mm, the dipping/cooking time is typically a couple of minutes to a few minutes. Our calculation gives around 3 mins for 20-mm diameter meatballs. For large meat balls (Lion Balls) or cubes (Pork Cube), they have diameters around 50 mm (2-inch), the typical cooking time in boiled soup is around 20 mins. Our calculation gives 16-20 mins.

Compare thermal diffusion with mass diffusion (salt, sugar, etc.)

- Physical parameters, including the thermal diffusivity (thermal diffusion coefficient), for some of our daily food materials.

Material properties	Unit	Pork	Chicken	Beef	Lamb	Fish	Shrimp	Bread	Egg	Turkey	Peking Duck	Pizza	Potato	Water
density	Kg/m3	1100	1150	1150	1150	1150	1150	800	1038	1050	1050	1200	1100	1000
specific heat	J/(Kg K)	3130	3500	3230	2800	3620	3650	2720	3000	3530	3000	2300	3670	4200
thermal conductivity	W/(m K)	0.45	0.45	0.48	0.5	0.54	0.5	0.5	0.58	0.5	0.45	0.5	0.55	0.58
thermal diffusivity	m2/s	1.31E-07	1.12E-07	1.29E-07	1.55E-07	1.30E-07	1.19E-07	2.30E-07	1.86E-07	1.35E-07	1.43E-07	1.81E-07	1.36E-07	1.38E-07

- However, the reported mass diffusion coefficients, D_M, is in the range from $1.0\text{x}10^{-12}$ m^2/s to $1.0\text{x}10^{-8}$ m^2/s, with the most credible reported values around $2\text{-}5\text{x}10^{-10}$ m^2/s, for diffusion of salt, sugar, water, etc. in the body of the typical foods.

Intelligence in Chinese Cooking

Both heating and salting times are quadratic power to the critical dimension of the food.
Cut the critical dimension by half will reduce the cooking time by 4 times.

- Thermal diffusivity ~ 1×10^{-7} m^2/s
- Salt diffusion coefficient ~ 1×10^{-9} m^2/s
- Salt diffuses 100 times slower than heat (still 10 times slower even at elevated temperature)

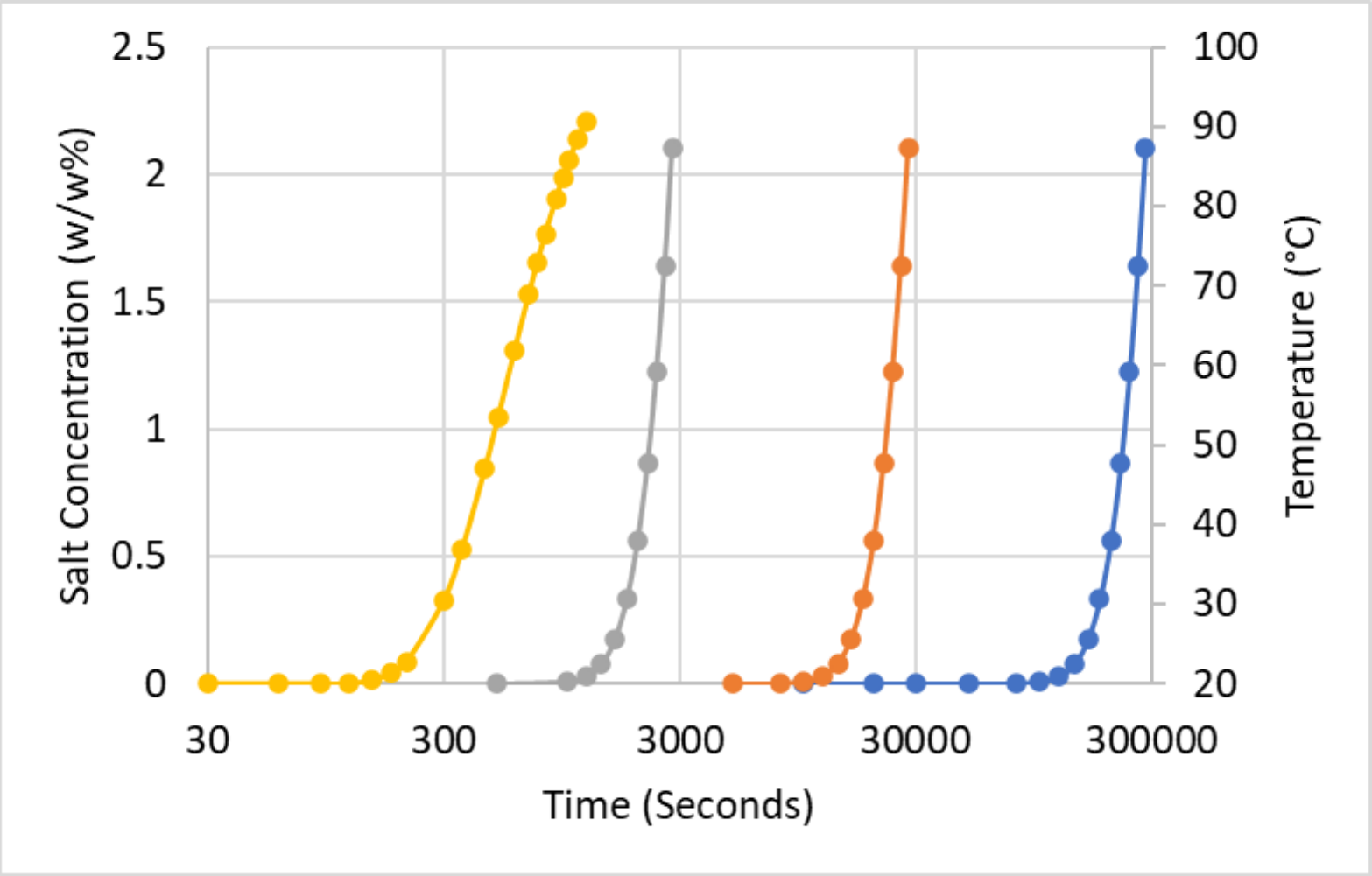
The fastest serving restaurant: 3 minutes

Pre-cut the food into small pieces (cubes, slices, strips, etc.) will significantly reduce the cooking time and meanwhile make the salting and other flavoring much more effective, by a factor of 10 to 100!

The famous "Spyce Kitchen", the robotic restaurant founded by four MIT graduates and Michelle 4-Star cook, claimed fast on-site cooking delivery with a cooking time of less than 3 minutes, is based on stir-frying cooking method with shredded food pieces.





The figure shows the calculated temperature and salt diffusion as a function of time for a spherical food piece with a diameter of 50 mm (for instance, a meatball). The yellow curve shows the center temperature rise with time during cooking with the thermal diffusivity of 1.5×10^{-7} m^2/s. The gray, orange, and blue curves are the calculated center salt concentration with time, with the salt diffusion coefficient of 1.5×10^{-10} m^2/s (blue), 1.5×10^{-9} m^2/s (orange), and 1.5×10^{-8} m^2/s (gray).

The Physics Behind Cooking Intelligence

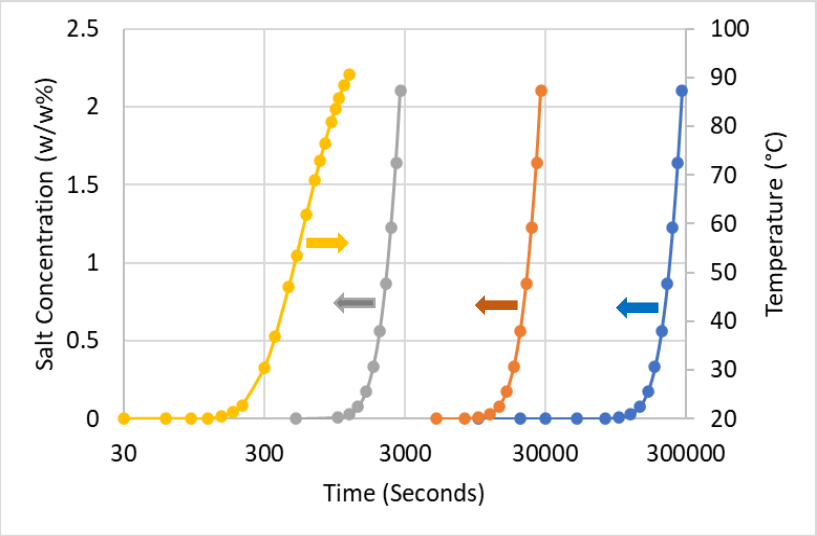
Both heating and salting times are quadratic power to the critical dimension of the food. Cut the critical dimension by half will reduce the cooking time by 4 times.

- ❑ Thermal diffusivity ~ 1×10^{-7} m^2/s
- ❑ Salt diffusion coefficient ~ 1×10^{-9} m^2/s
- ❑ Salt diffuses 100 times slower than heat (still 10 times slower even at elevated temperature)

The fastest serving restaurant: 3 minutes

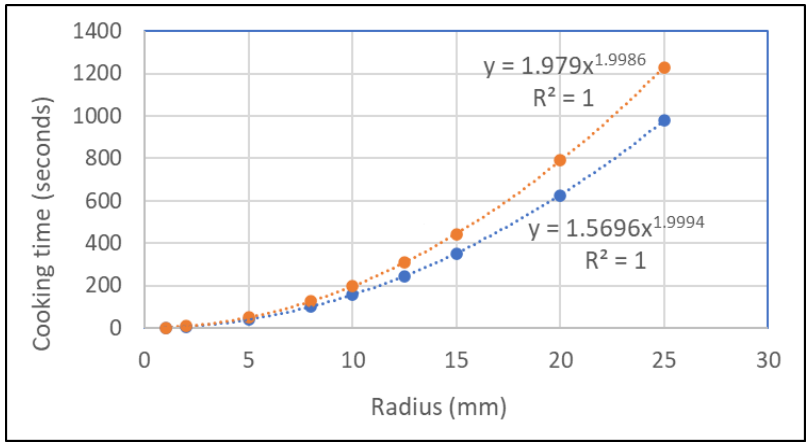
Pre-cut the food into small pieces (cubes, slices, strips, etc.) will significantly reduce the cooking time and meanwhile make the salting and other flavoring much more effective, by a factor of 10 to 100!

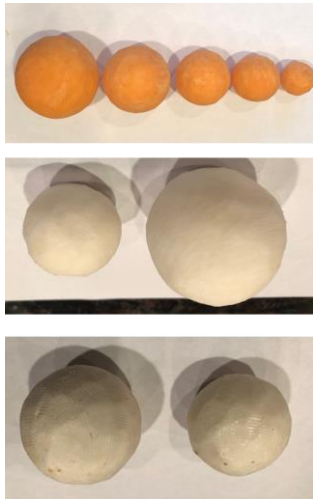
The famous "Spyce Kitchen", the robotic restaurant founded by four MIT graduates and Michelle 4-Star cook, claimed fast on-site cooking delivery with a cooking time of less than 3 minutes, is based on stir-frying cooking method with shredded food pieces.



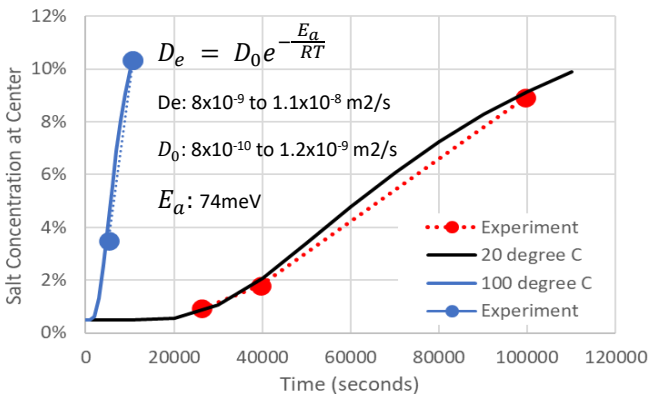
The calculated temperature and salt diffusion as a function of time for a spherical food piece with a diameter of 50 mm (for instance, a meatball). The yellow curve shows the center temperature rise with time during cooking with the thermal diffusivity of 1.5×10^{-7} m^2/s. The gray, orange and blue curves are the calculated center salt concentration with time, with the salt diffusion coefficient of 1.5×10^{-10} m^2/s (blue), 1.5×10^{-9} m^2/s (orange), and 1.5×10^{-8} m^2/s (gray).

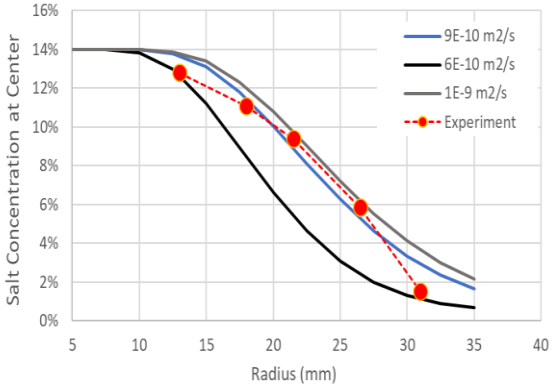
The Quadratic Dependence of Cooking Time to Food Critical Dimension





A Simple Approach to Determine Diffusion Coefficient of Salt in Various Food





$$\nabla \cdot \nabla C = \frac{1}{D}\frac{\partial C}{\partial t}$$

The concentration at the center of the sphere is (r = 0):

$$C_c = C_h - 2(C_h - C_0)\sum_{n=1}^{\infty}\left[(-1)^{n+1}\cdot e^{-t/\tau}\right] \qquad \text{Where: } \tau = \frac{R^2}{\pi^2 \cdot D}$$

Our previous publication:

Yifei "Jenny" Jin, Lisa R. Wang, and Jian Jim Wang, Physics in turkey cooking: Revisit the Panofsky formula, ***AIP Advances*** 11, 115316 (2021); https://doi.org/10.1063/5.0067811

Lisa R. Wang, Yifei "Jenny" Jin, and Jian Jim Wang, A Simple and Low-cost Experimental Method to Determine the Thermal Diffusivity of Various Types of Foods, ***American Journal of Physics***, Vol.90, Issue 8, https://doi.org/10.1119/5.0087135 DOI: 10.1119/5.0087135 August, 2022.

Additional Slides: Theoretical Model

Heat Transfer Equation

Fourier's Law states that the heat flux q (in W/m^2) is proportional to the temperature gradient, i.e., q = - k· $\frac{dT}{dx}$ for one-dimensional systems. For the 3-dimensional system, $\vec{q}$ = - k· **∇**T where $\vec{q}$ is a vector and **∇** is the gradient. k is thermal conductivity in W/(cm· K).

$$Q_{net} = A \cdot (q_{x+\delta x} - q_x) = -kA \cdot \left[\frac{\partial T}{\partial x}_{x+\delta x} - \frac{\partial T}{\partial x}_{x}\right] = -kA \cdot \left[\frac{\frac{\partial T}{\partial x}_{x+\delta x} - \frac{\partial T}{\partial x}_{x}}{dx}\right] \cdot dx = -kA \cdot \frac{\partial^2 T}{\partial x^2} \cdot dx$$

$$-Q_{net} = \frac{dU}{dt} = \rho c A \cdot \frac{d(T - T_{ref})}{dt} \cdot dx = \rho c A \cdot \frac{dT}{dt} \cdot dx$$

This leads to the one-dimensional heat diffusion equation:

$$\frac{\partial^2 T}{\partial x^2} = \frac{\rho c}{k}\frac{\partial T}{\partial t} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Where T = T (x, t) and α = k/ρc is the thermal diffusivity in m^2/s, where ρ is the density (kg/m^3) and c is the specific heat (J/(kg · K)).

In three-dimension, the heat transfer equation becomes:

$$\boxed{\mathbf{\nabla} \cdot \mathbf{\nabla} \mathrm{T} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

Where,

$$\mathbf{\nabla} \cdot \mathbf{\nabla} T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \qquad (for\ Cartisian\ coordinates)$$

$$= \frac{1}{r^2 \sin\theta}\left[\sin\theta \frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2 T}{\partial \varphi^2}\right] \qquad (for\ spherical\ coordinates)$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 T}{\partial \theta^2}\right) + \frac{\partial^2 T}{\partial z^2} \qquad (for\ cylinderical\ coordinates)$$

Heat Transfer for A sphere with azimuthal symmetry

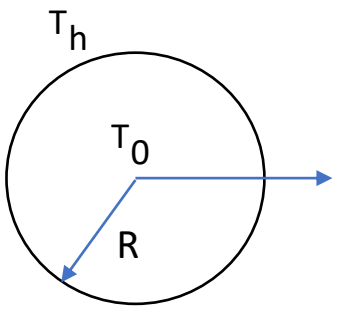
For a sphere with azimuthal symmetry, during the heat transfer, we have $\frac{\partial T}{\partial \theta} = 0$ and $\frac{\partial^2 T}{\partial \varphi^2} = 0$, the heat transfer equation becomes

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)\right] = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Applying $V = r \cdot T$ to the above equation, for $0 \leq r \leq R$ we get:

$$\frac{\partial^2 V}{\partial r^2} = \frac{1}{\alpha}\frac{\partial V}{\partial t}$$

We can decouple V(r, t) into:



$$V(r,t) = R(r) \cdot T(t)$$

And we get:

$$\frac{\partial V}{\partial t} = R(r) \cdot \frac{\partial T}{\partial t} = R(r) \cdot T'(t)$$

And:

$$\frac{\partial^2 V}{\partial r^2} = T(t) \cdot R''(r)$$

Then, we have:

$$T(t) \cdot R''(t) = \frac{1}{\alpha} \cdot R(r) \cdot T'(t)$$

It can be rearranged into:

$$\frac{R''(r)}{R(r)} = \frac{1}{\alpha} \cdot \frac{T'(t)}{T(t)}$$

Since the left side is only be r-dependent and the right side is only be t-dependent, and since they equal to each other, they must be neither r- or t- dependent. So, we have:

$$\frac{R''(r)}{R(r)} = \frac{1}{\alpha} \cdot \frac{T'(t)}{T(t)} = -\lambda$$

Then, we have:

$$R'' + \lambda R = 0$$

And

$$T' + \lambda\alpha T = 0$$

From the above equation, we have:

$$\frac{dT}{dt} = -\lambda\alpha T$$

$$\frac{dT}{T} = -\lambda\alpha \cdot dt$$

$$\int_0^t \frac{dT}{T} = -\lambda\alpha \cdot \int_0^t dt$$

$$lnT(t) - lnT(0) = -\lambda\alpha t$$

$$T(t) = e^{-\lambda\alpha t} \cdot T(0)$$

For $R'' + \lambda R = 0$

$$\frac{d^2R(r)}{dr^2} = -\lambda \cdot R(r)$$

$$R(r) = \mathrm{A}\cos\sqrt{\lambda}\cdot r + \mathrm{B}\sin\sqrt{\lambda}\cdot r$$

Now, we have:

$$V(r,t) = \sum_{\lambda}\left[\left(\mathrm{A}\cos\sqrt{\lambda}\cdot r + \mathrm{B}\sin\sqrt{\lambda}\cdot r\right)\cdot e^{-\lambda\alpha t}\right]$$

$$T(r,t) = \sum_{\lambda}\left[\left(\mathrm{A}\cos\sqrt{\lambda}\cdot r + \mathrm{B}\sin\sqrt{\lambda}\cdot r\right)\cdot \frac{e^{-\lambda\alpha t}}{r}\right]$$

For cooking (heating) food with food starting with low temperature T_0 and surrounded at high temperature (bath temperature) T_h, we have the following boundary conditions:

$T(r,o) = T_0\ (0 \le r \le R)$, where R is the radius of the sphere.

$$T(\ge R,t) = T_h$$

We have:

$A = 0, and\ \lambda = \left(\frac{n\pi}{R}\right)^2$ where $n = 1, 2, 3, \ldots$

We then have:

$$T(r,t) = T_h - \frac{2R(T_h - T_0)}{\pi\cdot r}\sum_{n=1}^{\infty}\left[\frac{(-1)^{n+1}}{n}\sin\frac{n\pi r}{R}\cdot e^{-\alpha n^2\pi^2 t/R^2}\right]$$

for $(0 \le r \le R)$

We define

$\tau = \frac{R^2}{\pi^2\cdot\alpha}$ as the time constant.

Thus, we have:

$$T(r,t) = T_h - \frac{2R(T_h - T_0)}{\pi\cdot r}\sum_{n=1}^{\infty}\left[\frac{(-1)^{n+1}}{n}\sin\frac{n\pi r}{R}\cdot e^{-t/\tau}\right]$$

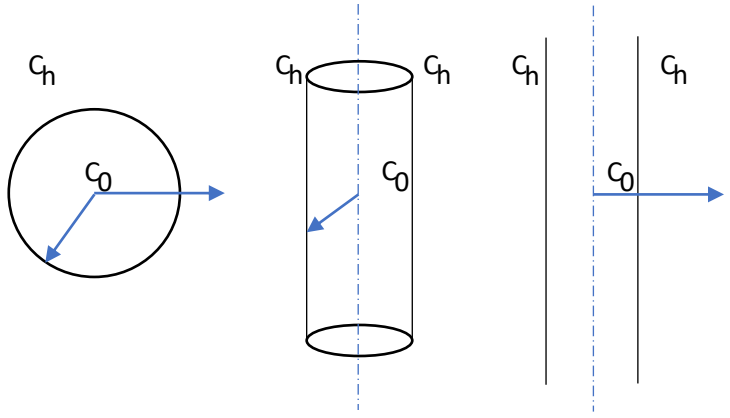
The temperature at the center of the sphere is (r = 0):

$$T_c = T_h - 2(T_h - T_0)\sum_{n=1}^{\infty}\left[(-1)^{n+1}\cdot e^{-t/\tau}\right]$$

We can spell out the equation with some of the initial (and deciding) terms:

$$T_c = T_h - 2(T_h - T_0)\{e^{-t/\tau} - e^{-4t/\tau} + e^{-9t/\tau} - e^{-16t/\tau} + e^{-25t/\tau} - e^{-36t/\tau} + e^{-49t/\tau} - \ldots\} \quad (1)$$

Where: $\tau = \frac{R^2}{\pi^2\cdot\alpha}$ and $\alpha = \frac{k}{\rho c}$



(a) Sphere (b) Cylinder (c) Plate

Fick's Law states [21-25] that the mass transfer (i.e., diffusion) equation follows

$$\frac{\partial^2 C}{\partial x^2} = \frac{1}{D}\frac{\partial C}{\partial t}$$

Where C = C (x, t) and D is the mass diffusion coefficient in m^2/s.

In three-dimension, the mass transfer equation becomes:

$$\nabla \cdot \nabla \mathrm{C} = \frac{1}{D}\frac{\partial C}{\partial t}$$

Where,

$$\nabla \cdot \nabla T = \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2} \qquad (for\ Cartisian\ coordinates)$$

$$= \frac{1}{r^2 \sin\theta}\left[\sin\theta \frac{\partial}{\partial r}\left(r^2 \frac{\partial C}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial C}{\partial \theta}\right) + \frac{1}{\sin\theta}\frac{\partial^2 C}{\partial \varphi^2}\right] \qquad (for\ spherical\ coordinates)$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 C}{\partial \theta^2}\right) + \frac{\partial^2 C}{\partial z^2} \qquad (for\ cylinderical\ coordinates)$$

A sphere with azimuthal symmetry

For a sphere with azimuthal symmetry, during the mass transfer [18, 2-25], we have $\frac{\partial C}{\partial \theta} = 0$ and $\frac{\partial^2 C}{\partial \varphi^2} = 0$, the mass transfer equation becomes

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2 \frac{\partial C}{\partial r}\right)\right] = \frac{1}{D}\frac{\partial C}{\partial t}$$

Applying $V = r \cdot T$ to the above equation, for $0 \leq r \leq R$ we get:

$$\frac{\partial^2 V}{\partial r^2} = \frac{1}{D}\frac{\partial V}{\partial t}$$

We can decouple V(r, t) into:

$$V(r,t) = R(r) \cdot T(t)$$

And we get:

$$\frac{\partial V}{\partial t} = R(r) \cdot \frac{\partial T}{\partial t} = R(r) \cdot T'(t)$$

And:

$$\frac{\partial^2 V}{\partial r^2} = T(t) \cdot R''(r)$$

Then we have:

$$T(t) \cdot R''(t) = \frac{1}{D} \cdot R(r) \cdot T'(t)$$

It can be rearranged into:

$$\frac{R''(r)}{R(r)} = \frac{1}{D} \cdot \frac{T'(t)}{T(t)}$$

Since the left side is only be r-dependent and the right side is only be t-dependent, and since they equal to each other, they must be neither r- or t- dependent. So, we have:

$$\frac{R''(r)}{R(r)} = \frac{1}{D} \cdot \frac{T'(t)}{T(t)} = -\lambda$$

Then we have:

$$R'' + \lambda R = 0$$

And

$$T' + \lambda D T = 0$$

From the above equation, we have:

$$\frac{dT}{dt} = -\lambda D T$$

$$\frac{dT}{T} = -\lambda D \cdot dt$$

$$\int_0^t \frac{dT}{T} = -\lambda D \cdot \int_0^t dt$$

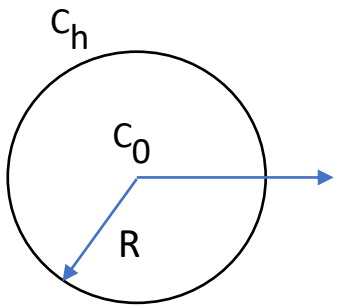
$$lnT(t) - lnT(0) = -\lambda D t$$

$$T(t) = e^{-\lambda D t} \cdot T(0)$$

For $R'' + \lambda R = 0$

$$\frac{d^2 R(r)}{dr^2} = -\lambda \cdot R(r)$$

$$R(r) = \mathrm{A}\cos\sqrt{\lambda} \cdot r + \mathrm{B}\sin\sqrt{\lambda} \cdot r$$



Now we have:

$$V(r,t) = \sum_{\lambda}\left[\left(\mathrm{A}\cos\sqrt{\lambda}\cdot r + \mathrm{B}\sin\sqrt{\lambda}\cdot r\right)\cdot e^{-\lambda Dt}\right]$$

$$T(r,t) = \sum_{\lambda}\left[\left(\mathrm{A}\cos\sqrt{\lambda}\cdot r + \mathrm{B}\sin\sqrt{\lambda}\cdot r\right)\cdot \frac{e^{-\lambda Dt}}{r}\right]$$

For brining (soaking) food with food starting with low concentration C_0 and surrounded at high concentration (bath concentration) C_h, we have the following boundary conditions:

$C(r,o) = C_0\ (0 \le r \le R)$, where R is the radius of the sphere.

$$C(\geq R,t) = C_h$$

We have:

$A = 0, and\ \lambda = \left(\frac{n\pi}{R}\right)^2$ where $n = 1, 2, 3, \ldots$

We then have:

$$C(r,t) = C_h - \frac{2R(C_h - C_0)}{\pi\cdot r}\sum_{n=1}^{\infty}\left[\frac{(-1)^{n+1}}{n}\sin\frac{n\pi r}{R}\cdot e^{-Dn^2\pi^2 t/R^2}\right]$$

for $(0 \le r \le R)$

We define

$\tau = \frac{R^2}{\pi^2\cdot D}$ as the time constant.

Thus, we have:

$$C(r,t) = C_h - \frac{2R(C_h - C_0)}{\pi\cdot r}\sum_{n=1}^{\infty}\left[\frac{(-1)^{n+1}}{n}\sin\frac{n\pi r}{R}\cdot e^{-t/\tau}\right]$$

The concentration at the center of the sphere is (r = 0):

$$C_c = C_h - 2(C_h - C_0)\sum_{n=1}^{\infty}\left[(-1)^{n+1} \cdot e^{-t/\tau}\right]$$

We can spell out the equation with some of the initial (and deciding) terms:

$$C_c = C_h - 2(C_h - C_0)\left\{e^{-t/\tau} - e^{-4t/\tau} + e^{-9t/\tau} - e^{-16t/\tau} + e^{-25t/\tau} - e^{-36t/\tau} + e^{-49t/\tau} - \ldots\right\}$$

(1)

Where: $\tau = \dfrac{R^2}{\pi^2 \cdot D}$