A Qualitative Comparison of the Fast Fourier Transform and the Morlet Wavelet Transform for Potential Depression Diagnosis

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ABSTRACT

The diagnosis of Major Depressive Disorder (i.e. "depression") has long relied on subjective measures, such as patient self-evaluation and doctor analysis. With few quantitative factors, this leaves depressed patients liable to be misdiagnosed and to not receive the necessary help, should they require it. To improve depression diagnosis, more quantitative methods must be explored. One such method is the analysis of electroencephalographic (EEG) data, which has a long history of usage in large-scale neurological assessment and, when taken from a resting-state patient, has potential applications in clinical settings. However, there are many different signal processing methods that can be used to extract features from EEG data. Thus, it is important to choose the most accurate method for depression-specific features. In this paper, two signal processing methods, the Fourier transform (FT) and the continuous wavelet transform (CWT), are used to analyze actual continuous, resting-state EEG data at well-established frequency bands to create similar topographic maps of a subject's brain. The transforms and topographic maps are created using the most recently updated MATLAB software as of September 9th, 2023, the 2023.0 version of the EEGLAB software, and the most recently updated FieldTrip toolbox software as of August 17th, 2023. The transforms' respective topographic maps are then assessed qualitatively to detect depression-specific outputs to ultimately determine the strengths of each transform in potential quantitative depression-diagnosis. Though both transforms have practical benefits, the paper finds that the CWT is ultimately stronger than the FFT at detecting depression biomarkers using this assessment method.

1. INTRODUCTION

1.1 Depression

About 3.8% of the world's population experiences some form of depression,[1] and about 9.7% of U.S. youth experience severe Major Depressive Disorder (MDD)[2]. In this paper, the more colloquial term "depression" is used interchangeably with the medical name of MDD. However, most modern depression diagnosis strategies, such as psychiatric evaluation by the *Diagnostic and Statistical Manual of Mental Disorders* (DSM-5) criteria,[12] rely on honest patient input and the doctor's analysis of the patient's perceived symptoms.[3] This system relies on little quantifiable data, leaving patients liable to be misdiagnosed or to have their mental health issues overlooked. Thus, depression diagnosis can be improved by examining data objectively to create a fairer method of depression diagnosis based on the individual. It is important that one chooses the optimal method of analysis to have the most accurate idea of the potential of the chosen diagnosis method. This paper assesses the effectiveness of two different signal processing methods, the Fourier transform and the wavelet transform, by examining electroencephalographic (EEG) data and considering the different factors that lend themselves to effective depression-focused EEG data analysis.

1.2 Fourier and Wavelet Signal Transforms

This section describes the two signal-processing techniques discussed in this paper. The first method, the discrete Fourier transform (FT), is a transform that converts a function into a form that describes the frequencies present in the original function (typically from a graph with time on the horizontal axis against frequency on the horizontal axis) by decomposing the original function into a sum of sine and cosine waves. See Fig. 2 for an example. The inverse discrete Fourier transform (IFT)

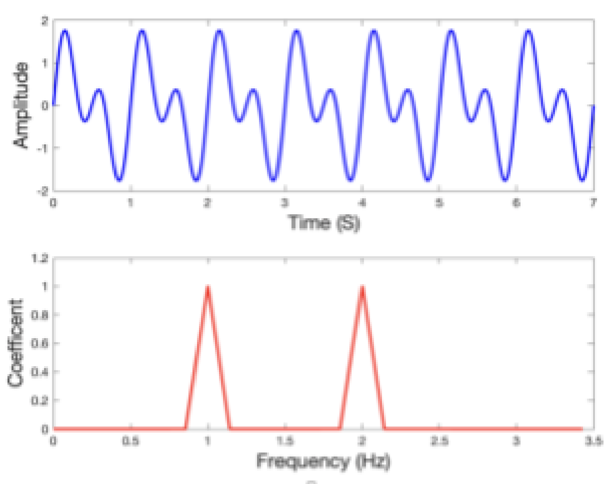


Figure 2: Top: a time series signal. Bottom: the signal transformed to Fourier (frequency) space.
Talebi, S. (2022a) Fourier vs. wavelet transform: What's the difference?, Built In. Available at: https://builtin.com/data-science/wavelet-transform (Accessed: 17 August 2023).

likewise converts from a frequency domain to a time domain. The fast Fourier transform (FFT) and inverse fast Fourier transform (IFFT) are methods for computing the discrete Fourier transform and its inverse efficiently.[7]

The second method used is the wavelet transform. A wavelet is a simple short-term oscillation with two important properties: scale and location. Scale is how stretched the wavelet is, and is correlated to frequency in Fourier transforms (see Fig. 3). Location is the location of the wavelet in time, since unlike waves, wavelets are only non-zero for a brief moment (see Fig. 4). The wavelet transform is used to compute how much of a wavelet is in the signal function for a particular scale and location by convolving it over the function.

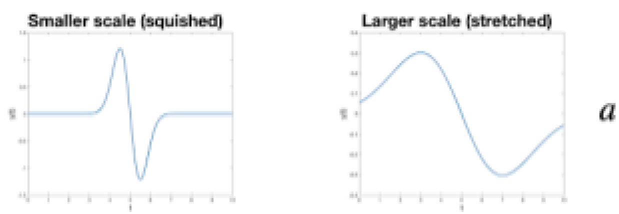


Figure 3: scale of Wavelet transform
Talebi, S. (2022a) Fourier vs. wavelet transform: What's the difference?, Built In. Available at: https://builtin.com/data-science/wavelet-transform (Accessed: 17 August 2023).

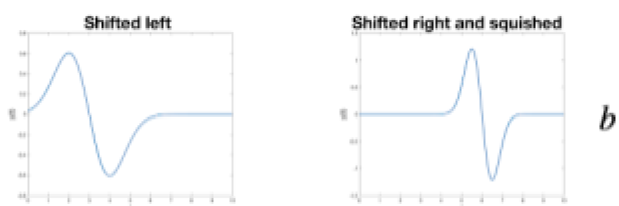


Figure 4: location of Wavelet transform
Talebi, S. (2022a) Fourier vs. wavelet transform: What's the difference?, Built In. Available at: https://builtin.com/data-science/wavelet-transform (Accessed: 17 August 2023).

To understand the difference between Fourier transforms and wavelet transforms, it is important to understand the Heisenberg Uncertainty Principle, which says that the degree of frequency spread and temporal spread cannot both be very small. In other words, if one wants to know the frequency of a signal to some significant degree of uncertainty, then at some point one cannot know the time at which the frequency occurs. The two transforms differ primarily in that with Fourier transforms, functions that are localized in the time domain have transforms that are spread out across the frequency domain and vice versa, so one knows either the frequency of or time at which a signal occurred perfectly, and the other property not at all. With wavelet transforms, however, the opposite case is true — that is, one can say the initial function had some particular amplitude variation that occurred during some identifiable times. This difference between the two transforms lends itself to interesting discussions when considering EEG data. Other more technical differences are described in Section 4.

The wavelet transform is often preferred over the pure Fourier transform due to its ability to capture time-varying frequency content and has historically been used in event-based EEG data analysis. However, resting-state EEG data for depression diagnosis should not vary significantly over the data collection period, given the absence of external triggers and the constancy of depressive disorder. As such, the juxtaposition of traditionally time-varying neurological data and the fixed nature of depression diagnosis makes an assessment of the strengths of these two well-known, widely used signal analysis techniques imperative.

2. ELECTROENCEPHALOGRAPHY (EEG)

2.1 Introduction to EEG data

In order to better understand how Fourier transforms and wavelet transforms are applied in the analysis of EEG data, a basic understanding regarding the qualities of EEG and EEG interpretation must first be established.

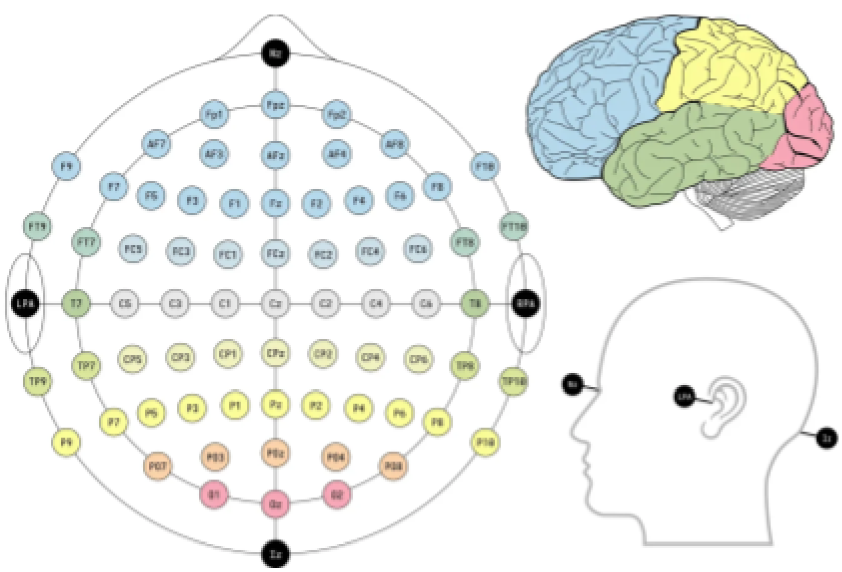


Figure 5: 10-10 electrode configuration diagram
Krol, L.R. (2020) File:EEG 10-10 system with additional information.svg, Wikimedia Commons. Available at: https://commons.wikimedia.org/wiki/File:EEG_10-10_system_with_additional_information.svg (Accessed: 31 August 2023).

EEG data is collected by attaching electrodes in some standardized configuration to a subject's scalp. One example of a commonly used electrode configuration, the 10-10 system, is shown in Fig. 5. The electrode locations are represented by the colored circles on the largest portion of the diagram. Each color electrode measures most directly the electrical signals in the respective same-colored part of the brain in the top-right corner of Fig. 5.

Electroencephalography (EEG) is a non-invasive method of recording an electrogram of spontaneous electrical activity in the brain. When collecting EEG data, electrodes are placed along the scalp as shown in Fig. 1.[4]

In depression diagnosis, EEG has been shown to differentiate patients with and without depression at a group-level, but its diagnostic potential on an individual level has yet to be realized. However, because quantitative EEGs produce complex data sets at multiple frequency bands, depending on electrode location and patient vigilance state (eyes open vs. closed), and because of its already established effectiveness on a group-level, the author believes EEG data analysis will be vital to depression diagnosis in the future.[5] In fact, the short-time Fourier transform, a variation of a technique discussed in this paper, has already been used to analyze EEG data and determine a correlation between speech data and depression.[6]

One example of EEG data is shown in Fig. 6. The bottom horizontal axis represents the time at which the data are recorded and is typically labeled in seconds. The vertical axis represents the electrode field measured by each electrode attached to the scalp, with each electrode label corresponding to a unique signal. The vertical lines in the example EEG data represent markers that show the timing of particular events, usually in cases where the observer would like to see the ways in which the EEG data change after some trigger or stimulus.[9]

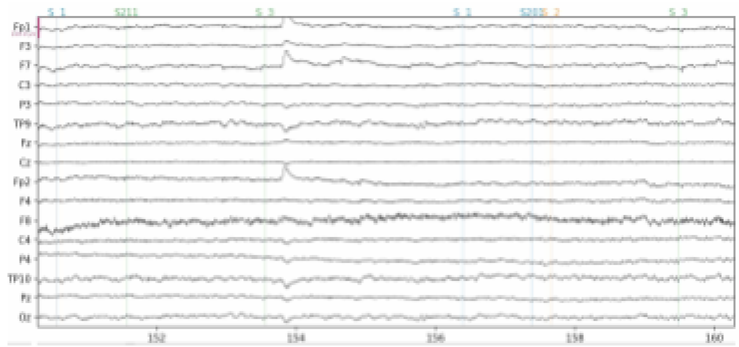


Figure 6: Example EEG data

Newman, A. J. (2020). *Introduction to EEG/ERP data*. Introduction to EEG/ERP Data - Data Science for Psychology and Neuroscience - in Python. https://neuraldatascience.io/7-eeg/introduction.html

The most important quality of EEG data is often considered to be its richness and variety. It is because of this richness that EEG data have the potential to be suitable to quantifiably diagnose depression. The biological mechanisms of depression are still relatively unknown, so it is prudent to begin quantifying diagnosis on a broader, whole-brain scale, rather than arbitrarily analyzing very specific parts of the brain. Once further research has been done relating depression diagnosis to specific areas of EEG and specific EEG frequency bands, researchers and doctors may want to look into the use of advanced computational methods (such as generative machine-learning algorithms), more specific but potentially more invasive neuroimaging techniques, or different data collection processes (e.g. fMRI, PET). Ultimately, the best solution for depression diagnosis may be one part a quantifiable combination of different technologies and data types, and another part still subjective self-assessment.

2.2 Interpretation of EEG data

When analyzing EEG outputs, one method professionals choose is to inspect historically pre-defined frequency bands in the resultant power spectrum that are typically referred to as alpha, beta, gamma, theta and delta waves.[10] The strength of these predefined frequency bands as a biomarker for various neurological states has been established and confirmed in the past.[10]

Figure 8 shows a diagram of the major exterior regions of the human brain, with the prefrontal cortex (left-hand side of Fig. 8) corresponding to the top of the topoplot and the front of the subject's scalp, and the occipital lobe (right-hand side of Fig. 8) corresponding to the bottom of the topoplot and the back of the subject's scalp. The correlation between the topoplot and the portions of the brain leads to intuitive analysis. For example, one may notice higher powers (red tones) in the lower portion of the topoplot above, signifying a higher presence of the specific frequency band being analyzed in the occipital lobe. This visual analysis is used in Section 4, in which a real dataset is analyzed.

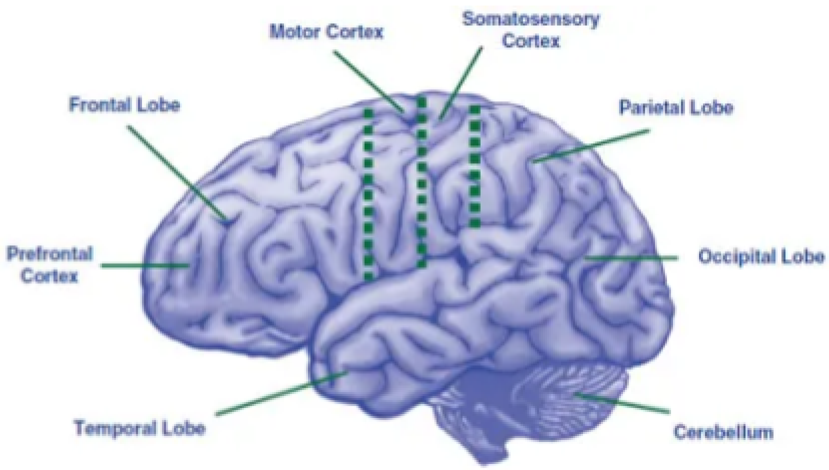


Figure 8: Major exterior regions of the brain, labeled Sousa, D. (2011) 6 major parts of the brain and how they work, How the Brain Learns: The Blog. Available at: https://howthebrainlearns.wordpress.com/2011/11/28/6-major-parts-of-the-brain-and-how-they-work/ (Accessed: 30 August 2023).

3. SIGNAL ANALYSIS

3.1 Introduction to signal analysis in EEG

When considering EEG data, signal analysis is most prominently used in the data preprocessing and feature extraction steps of EEG analysis. In feature extraction, signal analysis techniques are used to convert the complex EEG electrode data into a more suitable format for the purpose of analysis. Because of the many forms a "more suitable format" can take, and because of the many aspects of EEG data one may choose to focus on, feature extraction is a much more subjective area of signal analysis, and is thus the main focus of this paper.[11]

3.2 Fourier transform (FT)

3.2.1 Math

The Fourier transform (FT) is a mathematical technique that transforms a function in the time-domain to a function in the frequency-domain. The inverse Fourier transform (IFT) does the opposite, transforming a function of frequency to a function of time. In this paper, the Fourier transform is used interchangeably with the fast Fourier transform (FFT), as the FFT is just a computationally faster method of performing the same transform. Likewise, the IFT and the inverse fast Fourier transform (IFFT) are used interchangeably as well.

This domain-transform is achieved by decomposing the original time-domain function, $g(t)$, into a sum of sine and cosine waves of different frequencies, then plotting the strength of each present frequency as a frequency-domain function, $G(f)$.

Mathematically, the continuous FT is represented as:

1) $$G(f) = \int_{-\infty}^{\infty} g(t) \cdot e^{-2\pi i f t} dt$$

And the IFT is represented as:

2) $$g(t) = \int_{-\infty}^{\infty} G(f) \cdot e^{2\pi i f t} df$$

In the equations, $G(f)$ represents the frequency-domain representation of the signal, $g(t)$ represents the time-domain representation of the signal, f is the frequency at which the signal is being analyzed, and i is $\sqrt{-1}$. The FT and IFT output information in two dimensions, typically with either distinct frequency or time values on the horizontal axis and power on the vertical axis.

Note the periodic nature of the sine and cosine waves into which the Fourier transform decomposes the original function. The Fourier transform assumes a periodic nature to the original function as well, making it unsuitable to construct a non-periodic function from a small sample of the function's outputs. Despite this, the usefulness of Fourier transforms in analyzing periodic or entire sets of data is evident.

3.3 Wavelet transform

3.3.1 Math

The wavelet transform (WT) is a mathematical tool that, like the FT, facilitates the analysis of signals by revealing their short-term frequency characteristics. There exists an inverse wavelet transform (IWT) as well.

Whereas the FT decomposes a signal into its constituent sinusoidal components, the WT decomposes it into a set of wavelets. Wavelets are small, localized, oscillatory functions. One example of a wavelet, the Morlet wavelet, is shown in Fig. 11.

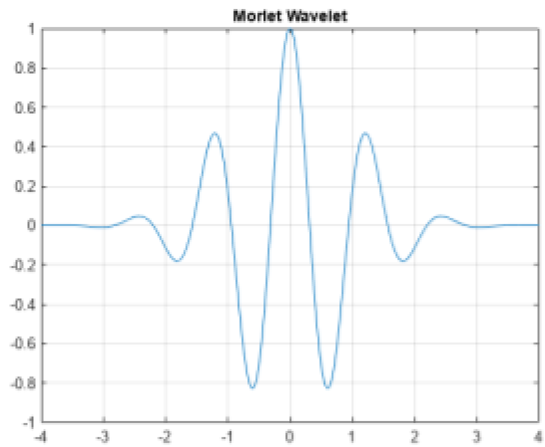


Figure 11: Morlet wavelet
Morlet wavelet - MATLAB Morlet (2006) MathWorks. Available at: https://www.mathworks.com/help/wavelet/ref/morlet.html (Accessed: 31 August 2023).

In this paper, the WT is used interchangeably with the continuous wavelet transform (CWT). Likewise, the IWT and the inverse continuous wavelet transform (ICWT) are used interchangeably as well.

The CWT involves a process known as convolution, in which the wavelet is compared in small segments of time across the entire signal, as in Fig. 12, then scaled and shifted to search for a match to the signal at that particular position.

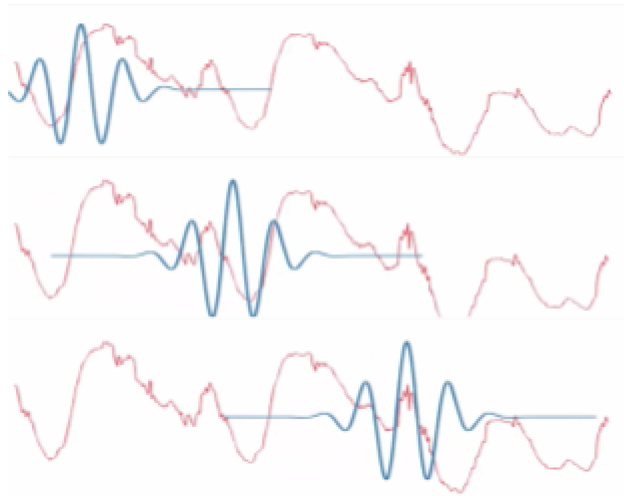


Figure 12: Wavelet is convolved over the original function
Wavelet transforms in Matlab (no date) Wavelet Transforms in MATLAB - MATLAB & Simulink. Available at: https://www.mathworks.com/discovery/wavelet-transforms.html (Accessed: 31 August 2023).

Mathematically, the CWT is represented as:

3) $$W(a,b) = \int g(t) \cdot \psi^*(\frac{t-b}{a})dt$$

And the ICWT is represented as:

4) $$g(t) = \frac{1}{C_\psi} \int W(a,b) \cdot \psi(\frac{t-b}{a}) \cdot \frac{da}{a^2}$$

In these equations, $W(a,b)$ is the CWT coefficient obtained from the wavelet transform, $g(t)$ is the time-domain representation of the signal, $\psi(t)$ and $\psi^*(t)$ are the function that describes the wavelet and its conjugate, respectively, a and b are the scale and translation parameters, respectively, and C_ψ is a constant that depends on the wavelet form used and normalizes the signal in the inverse transform. The value that represents the similarity of the wavelet with the original function scaled by a and at time b is represented by the right hand side of Eq. 3, $W(a,b)$. Thus, the CWT outputs information in three dimensions, typically with time values on the horizontal axis, scale on the vertical axis, and strength of correlation represented by different colors. Note that there is less of an obvious connection between the CWT and the ICWT than there is between the FFT and the IFFT, as the CWT and ICWT transform between different numbers of data dimensions.

3.4 Preliminary comparison of techniques from mathematical and practical application

The most notable difference between the FFT and the CWT is the different domains and scales on which the two methods output data. This is the most crucial difference in actual data analysis, as it controls what features of the original data are focused on and how they are translated to frequency information. Because this difference is difficult to assess purely theoretically, it will be assessed in greater depth in Section 4 with an actual EEG dataset.

From a more practical perspective, the FFT may be easier to use than the CWT due to its simplicity and ubiquity. Though the CWT is being used increasingly often, the FFT nonetheless has a longer history of use and has more literature regarding its usage.

The FFT is also computationally more efficient than the CWT because it analyzes the entire signal, rather than convolving over multiple scales like the CWT. This difference may be relevant in the future, should depression diagnosis with EEG data become significantly more widespread.

4. COMPARISON OF FFT AND CWT ON EXPERIMENTAL EEG DATA

4.1 Introduction to the dataset

In this section, a visual comparison of Fourier transforms and wavelet transforms is demonstrated. In order to do so, both Fourier analysis and wavelet analysis are used to generate a topographic plot (topoplot) of the brain of one subject who experiences recurrent episodes of MDD from the power spectra of preprocessed resting-state eyes-closed EEG data taken from the subject. These data are from the article, "A mind-brain-body dataset of MRI, EEG, cognition, emotion, and peripheral physiology in young and old adults".[13] The anonymous subject is labeled as subject 010044. The presence of Major Depressive Disorder, as well as other patient characteristics, can be confirmed by the meta file containing patient specification data. The whole study these data was taken from is largely unrelated to the purposes of this analysis, but for further information regarding the study, the methods of data collection, and the methods of data preprocessing, one can visit the original source, which describes these characteristics in great detail.[13]

The purpose of this analysis is to visually compare the efficacy of Fourier transforms and wavelet transforms to confirm the previously established mathematical benefits and shortcomings of each transform type, as well as to compare the two analysis methods more specifically in the analysis of depression-focused EEG data with the purpose of improving depression diagnosis. Specifically, Morlet wavelets are used, given their long standing use in time-frequency decomposition, especially with EEG data. Given the continuous, resting-state nature of these data, both the continuous FT and the CWT are appropriate to use in this analysis.

4.2 Methodology

The full code used to generate the topoplots shown in Section 4.4 can be found in Section 7. This code is written in the most recent version of the MATLAB software as of September 9th, 2023, using the 2023.0 version of the EEGLAB software and the FieldTrip toolbox most recently updated on August 17th, 2023.

The resultant topoplots from the Fourier and wavelet analysis are solely analyzed qualitatively, based on pre-established knowledge regarding the correlation between the strength of specific frequency bands in certain parts of the brain and the existence of MDD. As such, the power levels corresponding to the FFT- and CWT-generated graphs visualized in Section 4.4 are not modified to both use frequency, as the examples are in Section 3, nor are the graphs adjusted to account for the difference in the power level magnitudes (e.g. normalizing or plotting both sets of power values on a logarithmic scale). From the author's analysis, the visual outputs of scale-focused, frequency-focused, and normalized CWT and FFT graphs are all very comparable, thus the difference in power levels should not affect the qualitative analysis in this paper. More discussion surrounding the challenges in quantitative analysis specifically with this project can be found in Section 5.2. Nonetheless, additional code written by the author to output normalized, frequency-focused topoplots using both transforms is available in Section 7.

4.3 Output with FFT versus output with CWT

Figure 14 shows the topographic maps of the overall power spectra over all frequencies. The topoplot on the left uses FFT, while the topoplot on the right uses CWT.

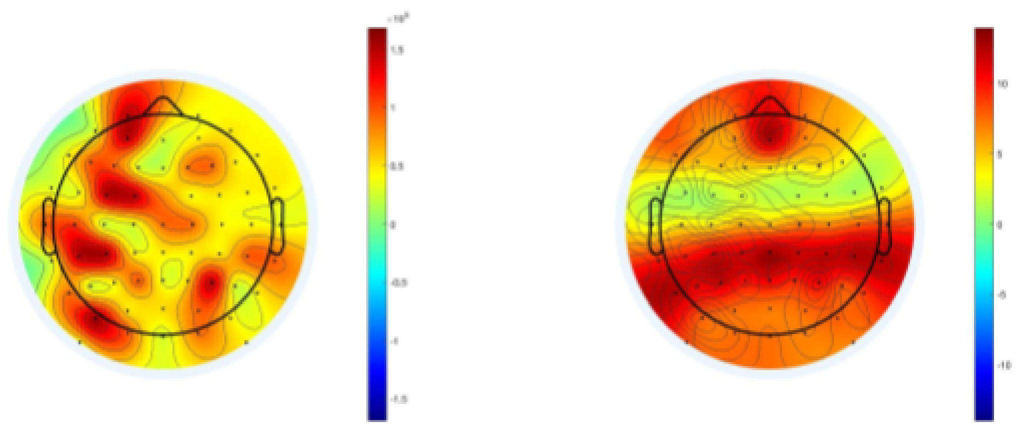
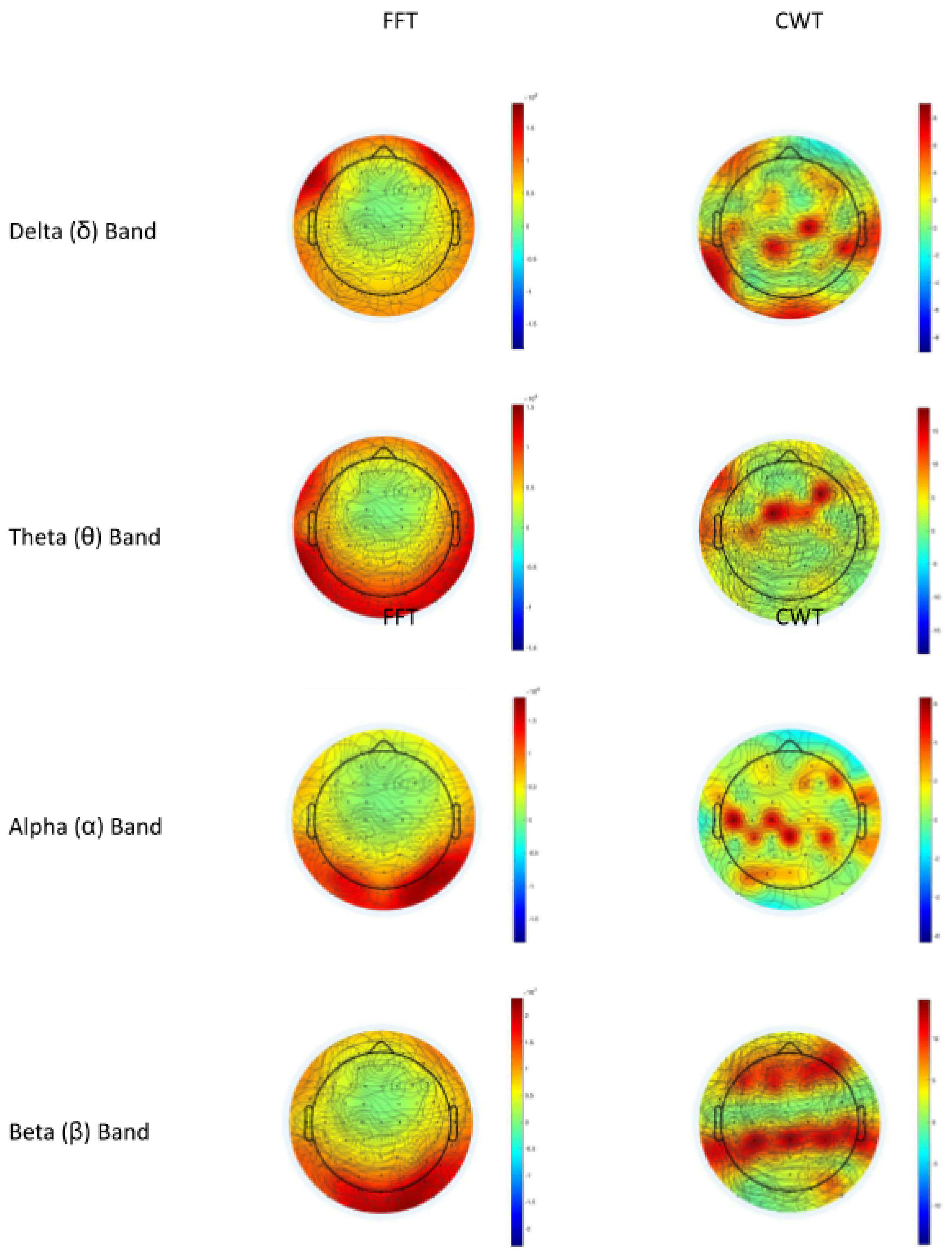


Figure 14: Left: topographic map of average power spectrum in depressed patient created using FFT analysis; right: topographic map of average power spectrum in depressed patient created using CWT analysis (Generated by author)

Figure 15 shows topographic maps of the power spectra in specific frequency bands, again with the FFT-generated topoplot on the left, the CWT-generated topoplot on the right, and the frequency band being analyzed specified on the left of both topoplots in each row.



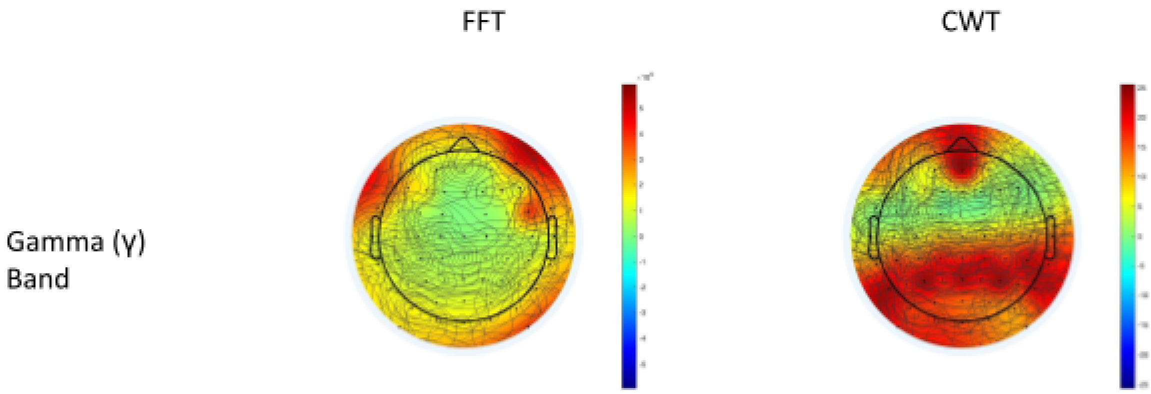


Figure 15: Topoplots for specific frequency bands, with band name specific on the left. Left: topoplot created using FFT analysis; right: topoplot created using CWT analysis (Generated by author)

4.4 Qualitative comparison and further analysis of signal processing techniques

There are some visually evident differences between the two outputs. Some of these differences are indicative of the potential benefits and shortcomings of Fourier transforms compared to wavelet transforms.

First, there are some visual differences that are non-unique to EEG data or the presence of MDD. Some of the difference between the two topographic plots can be attributed to the different ways in which FFT and CWT process temporal variability. Due to the unchanging nature of FTs, Fourier-based topographic maps provide a static view of power distribution across frequency bands. In other words, they do not capture how the data may change over time. On the contrary, wavelet topographic maps depict the potential time-varying nature of the power distribution, meaning that one may observe some short-term oscillatory patterns only present in the wavelet plot, creating more variation over the whole wavelet plot. This point is not as relevant for resting-state EEG data analysis, as there is no reason for the data to change over time in this scenario, but in the analysis of how depressive brain activity changes (e.g. after applying different triggers), this is one important benefit of CWTs over FFTs.

The CWT plot is also more spatially distributed than the FFT plot, as can be seen in its higher number of peaks, especially in the delta, theta, and alpha bands. This indicates that EEG power reads as more spread out over multiple brain regions with CWT analysis. This aligns with one's expectations, as established in Section 3.4. Fourier-based maps focus on power at specific frequencies, whereas CWT maps can show broader patterns of power distribution across both frequencies and time.

Somewhat surprisingly, no difference in noise and artifact sensitivity can be seen in the two plots. Because Fourier-based maps consider all data points simultaneously, one would expect the Fourier-generated topoplot to be affected by noise and artifacts to a greater degree than the wavelet-based topoplot. However, this is not the case, as there are no unexpected frequency spikes in either set of topoplots. It may be the case that this dataset is too clean to effectively assess the degree to which different transform-generated topoplots are affected by noise and artifacts.

With regards to depression diagnosis specifically, one method of analyzing the strength of each transform is observing dominant frequency bands that correspond to specific brain regions that correlate to the presence of MDD. To understand the locations of the brain regions specified in the following analysis, one can refer back to Fig. 8 in Section 2.2.

In the delta band, which ranges from 0.5-4 Hz, higher delta power in frontal regions has been correlated with the presence of MDD.[14] This higher delta power can be seen in Fig. 15 in the CWT plot. The frontal lobe area has a zero or near-zero power level in the FFT plot of the delta band.

In the theta band, which ranges from 4-8 Hz, higher theta power in frontal regions has been correlated with the presence of MDD.[14] This higher theta power can be seen in Fig. 15 in the CWT plot. The frontal lobe area has a zero or near-zero power level in the FFT plot of the theta band.

In the alpha band, which ranges from 8-13 Hz, greater alpha asymmetry and higher alpha power in occipital regions has been correlated with the presence of MDD.[15, 16, 17] With regard to alpha asymmetry, the CWT plot is clearly more asymmetric than the FFT plot. However, with regard to alpha power, higher power is most evident in the FFT-generated topoplot, with a high concentration of alpha-band frequencies in the back portion of the brain. Some higher alpha power in the occipital region is visible in the CWT-generated topoplot as well, but not to the degree seen in the FFT plot.

In the beta band, which ranges from 13-30 Hz, an overall increase in absolute beta power has been correlated with the presence of MDD.[14] Because an overall increase in power would affect all parts of the resultant topoplot similarly, rather than causing higher power levels in certain regions, it cannot be assessed through qualitative analysis of the topoplots in Fig. 15. Thus, the author chooses to disregard the beta band in this analysis.

In the gamma band, which includes all frequencies above 30 Hz, reduced gamma power in the anterior cingulate cortex (see Fig. 16) and increased resting complexity of gamma signaling in the frontal and parietal cortex have been correlated with the presence of MDD.[18, 19] Reduced gamma power in the anterior cingulate cortex relative to other regions of the brain is most clearly seen in the green (low-power) area slightly above the center of the topoplot in the CWT graph. Neither the FFT nor the CWT plots seems to show significant increased resting complexity of gamma signaling in the frontal or parietal cortices, as the resultant plot is fairly smooth and homogenous.

Figure 16: Diagram of the anterior cingulate cortex as viewed from a subject's left-hand side, with the subject's nose on the viewer's left and the back of the subject's head on the viewer's right.

Hall. (2011, September 8). MRI cingulate cortex. Wikimedia Commons. https://commons.wikimedia.org/wiki/File:MRI_cingulate_cortex.p

5. CONCLUSION AND FURTHER RESEARCH

5.1 Conclusion

As a whole, both the Fourier transform and the wavelet transform have their respective strengths and weaknesses in the analysis of EEG data for the detection of Major Depressive Disorder. The FFT has traditionally been used to analyze the type of continuous, resting-state data one is likely to collect in a clinical setting and has a longer precedent of use than the CWT. However, in the analysis of typically non-periodic, complex EEG data specifically, the CWT has recently seen more use due to its ability to output both time and frequency information simultaneously. Nonetheless, the most important practical difference between the FFT and the CWT stems from this ability. CWT outputs are typically more spatially distributed than FFT outputs, which can lead to CWT outputs being less precise. In this paper, the greater spatial distribution of the CWT was not particularly detrimental to the subsequent analysis of the CWT output. Thus, each type of signal processing technique has its own historically- and practically-based benefits.

Moreover, to the degree analyzed by this paper, the CWT is generally stronger than the FFT at detecting depression biomarkers. In both the delta and theta frequency bands, the CWT is somewhat stronger at detecting higher frontal region power levels, which are indicative of depression in these two frequency bands. In the alpha band, the CWT is somewhat stronger at displaying asymmetry in power levels over the whole brain, but the FFT is somewhat stronger at detecting higher power levels in the occipital region of the brain. In the gamma band, the CWT is significantly stronger at detecting lower power levels in the anterior cingulate cortex. The beta band is excluded from this analysis because of its difficulty of analysis using purely qualitative methods.

5.2 Further research

There is substantial room for further research in both the breadth and depth of this paper's findings, as well as in the ways in which EEG can be used to further objective depression diagnosis efforts.

This paper compares two signal-processing techniques, Fourier transforms and Morelet wavelet transforms. Other signal-processing methods, such as the spectrogram, Hilbert-Huang Transform, and time-frequency decomposition techniques should be explored and detailed, so that experimenters deciding which signal-analysis method to use will truly be able to choose the most optimal for the purposes of their experiment.

The paper is also limited by the data analyzed in Section 4. Most notably, because the only individual in the dataset with both MDD and good-quality data also had relatively high levels of recent alcohol consumption, the resultant EEG data (and topographic map) may be confounded by this factor. More broadly, because only one individual was analyzed, their analysis may not necessarily reflect the typical analysis of EEG data with FFTs and wavelet transforms. Using more data points would establish a multi-faceted understanding of both the qualities of the two transforms as well as the way in which depression can be considered in the analysis of a resultant topographic plot graph.

The analysis in this paper is also qualitative rather than quantitative. One aspect of the project that makes quantitative analysis challenging is the difficulty of translating both the FFT-transformed output and the CWT-transformed output to both be in terms of frequency and to display comparable power levels. Although the CWT output can be put in terms of frequency rather than wavelet scale and normalized, as can be referenced in the code in Section 7, the magnitude of the power levels displayed in the CWT topoplots is still somewhat smaller than the magnitude of the power levels displayed in the FFT topoplots. These variations in power scale are to be expected. Fundamentally, there are many factors that affect the power levels of the outputs of the two transforms, some being the choice of scales in the CWT analysis, the number of frequency bins in the FFT analysis, and the choice of wavelet for the CWT analysis.

Regardless, it is true that pure qualitative analysis may lead to imprecise comparisons. However, visual analysis is sufficient to achieve the purpose of this paper, which is to understand the differences between the FFT and the CWT in EEG signal analysis specifically for depression diagnosis. In the future, the author plans to improve their findings by quantifying the differences between FFT and CWT through statistical analysis. One method may be to analyze both healthy and depressed datasets and quantify the significance between the representations with both FFT and CWT.

With regard to the future of EEG data in depression analysis as a whole, more exact data collection methods could be used to assess specific areas of the brain. Some examples of these data collection methods may include the more invasive intracranial EEG (iEEG) and a combination of EEG and function magnetic resonance imaging (fMRI) known as simultaneous EEG-fMRI. Similarly, advanced algorithmic methods may be used to strengthen the correlation between certain data and the presence of MDD. For example, generative AI could be used to assess depressive disorder and some form of EEG data on an individual level, especially given the idiosyncratic nature of the brain. Eventually, the author hopes to explore the feasibility of real-time EEG-based depression monitoring and its practical implications for clinical settings.

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7, APPENDIX

The original code the author uses to generate the images shown in the paper is shown below:

For the overall analysis with the FFT:

```
# load EEGLAB and FieldTrip (adjust the paths as needed)
cd '[insert EEGLAB path]'
eeglab
cd '[insert Fieldtrip path]'
ft_defaults;

# load the data (pre-processing is done beforehand)
EEG = pop_loadset('filename', '[insert file name]', 'filepath', '[insert file path]'); % adjust dataset and path name as needed
EEGData = EEG.data;

# define the Fourier transform parameters
N = size(EEGData, 2); # number of data points
fs = EEG.srate; # sampling rate
frequencies = 0:fs/N:(fs/2); # frequency values for plotting (up to Nyquist frequency)

fftData = fft(EEGData, [], 2); # calculate the Fourier transform of EEG data
powerSpectrum = abs(fftData).^2; # calculate the power spectrum (absolute squared values)
avgPowerSpectrum = mean(powerSpectrum, 1); # calculate the average power across all channels

topoplot(avgPowerSpectrum, EEG.chanlocs, 'electrodes', 'on'); # plot the topographic map of the average power spectrum
colorbar;
```

For the overall analysis with the CWT:

```
# load EEGLAB and FieldTrip (adjust the paths as needed)
cd '[insert EEGLAB path]'
eeglab
cd '[insert Fieldtrip path]'
ft_defaults;

# load the data (pre-processing is done beforehand)
EEG = pop_loadset('filename', '[insert file name]', 'filepath', '[insert file path]'); % adjust dataset and path name as needed
EEGData = EEG.data;

# define the CWT parameters.
wavelet = 'morl'; # Morlet wavelet (commonly used in EEG analysis)
scales = 1:0.5:50; # scale range for the wavelet transform
fs = EEG.srate; # sampling rate

# perform the continuous wavelet transform (CWT).
cwt_data = cwt(EEGData, scales, wavelet);

# calculate the power of the CWT coefficients (squared magnitude).
power_cwt = abs(cwt_data).^2;

# calculate the average power across all channels
avg_power_cwt = mean(power_cwt, 1);

# plot the topographic map of the average CWT power.
topoplot(avg_power_cwt, EEG.chanlocs, 'electrodes', 'on');
colorbar;
```

For the analysis at a specific frequency band with the FFT:

```
# This is the code for the alpha band; adjust as necessary for the other four frequency bands

# load EEGLAB and FieldTrip (adjust the paths as needed)
cd '[insert EEGLAB path]'
eeglab
cd '[insert Fieldtrip path]'
ft_defaults;

# load the data (pre-processing is done beforehand)
EEG = pop_loadset('filename', '[insert file name]', 'filepath', '[insert file path]'); % adjust dataset and path name as needed
EEGData = EEG.data;

# define the Fourier transform parameters
N = size(EEGData, 2); # number of data points
fs = EEG.srate; # sampling rate
frequencies = 0:fs/N:(fs/2); # frequency values for plotting (up to Nyquist frequency)

fftData = fft(EEGData, [], 2); # calculate the Fourier transform of EEG data
powerSpectrum = abs(fftData).^2; # calculate the power spectrum (absolute squared values)

alphaBand = [8 13]; # alpha band (8-13 Hz) # define the frequency bands of interest (adjust as needed)
alphaBandIndices = find(frequencies >= alphaBand(1) & frequencies <= alphaBand(2)); # find the indices corresponding to the alpha band frequencies
alphaPower = mean(powerSpectrum(:, alphaBandIndices), 2); # Calculate the average power within the alpha band for each channel

topoplot(alphaPower, EEG.chanlocs, 'electrodes', 'on'); # Plot the topographic map of average alpha power
colorbar;
```

For the analysis at a specific frequency band with the CWT, in terms of scale:

```
# This is the code for the alpha band; adjust as necessary for the other four frequency bands

# load EEGLAB and FieldTrip (adjust the paths as needed)
cd '[insert EEGLAB path]'
eeglab
cd '[insert Fieldtrip path]'
ft_defaults;

# load the data (pre-processing is done beforehand)
EEG = pop_loadset('filename', '[insert file name]', 'filepath', '[insert file path]'); % adjust dataset and path name as needed
EEGData = EEG.data;

# define the CWT parameters.
wavelet = 'morl'; # Morlet wavelet (commonly used in EEG analysis)
scales = 1:0.5:50; # scale range for the wavelet transform
fs = EEG.srate; # sampling rate

# perform the continuous wavelet transform (CWT).
cwt_data = cwt(EEGData, scales, wavelet);

# calculate the power of the CWT coefficients (squared magnitude).
power_cwt = abs(cwt_data).^2;

# define the frequency bands of interest (adjust as necessary)
alpha_band = [8 13];

```

```
# calculate the average power within the alpha band for each channel.
alpha_power = zeros(1, size(power_cwt, 2)); # preallocate for average alpha power

for ch = 1:size(power_cwt, 2)
    # extract the CWT coefficients for the current channel.
    cwt_channel = power_cwt(:, ch);

    # calculate the average power within the alpha band.
    alpha_power(ch) = mean(cwt_channel(scales >= alpha_band(1) & scales <= alpha_band(2)));
end

# plot the topographic map of average alpha power.
topoplot(alpha_power, EEG.chanlocs, 'electrodes', 'on');
colorbar;
```

For the normalized analysis at a specific frequency band with the CWT, when its scale-outputs are converted to frequency:

```
# Load the data (pre-processing is done beforehand)
EEG = pop_loadset('filename', '[insert file name]', 'filepath', '[insert file path]'); # Adjust dataset and path name as needed
EEGData = EEG.data;

# Define the CWT parameters.
wavelet = 'morl'; # Morlet wavelet (commonly used in EEG analysis)
scales = 1:0.5:50; # Scale range for the wavelet transform
fs = EEG.srate; # Sampling rate

# Perform the continuous wavelet transform (CWT).
cwt_data = cwt(EEGData, scales, wavelet);

# Define the frequency values corresponding to each scale.
frequencies = scal2frq(scales, wavelet, 1/fs);

# Define the alpha band (8-13 Hz) indices.
alpha_band_indices = find(frequencies >= 8 & frequencies <= 13);

# Extract CWT coefficients within the alpha band.
cwt_alpha_band = cwt_data(alpha_band_indices, :);

# Calculate the power of the CWT coefficients within the alpha band (squared magnitude).
power_alpha_band = abs(cwt_alpha_band).^2;

# Calculate the average power within the alpha band for each channel.
avg_power_alpha_band = mean(power_alpha_band, 1);

# Normalize the power values for better comparability.
normalized_avg_power_alpha_band = (avg_power_alpha_band - min(avg_power_alpha_band)) / (max(avg_power_alpha_band) - min(avg_power_alpha_band)

# Plot the topographic map of the normalized CWT power within the alpha band.
topoplot(normalized_avg_power_alpha_band, EEG.chanlocs, 'electrodes', 'on');
colorbar;
title('Normalized CWT Power within Alpha Band (8-13 Hz)');
```

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